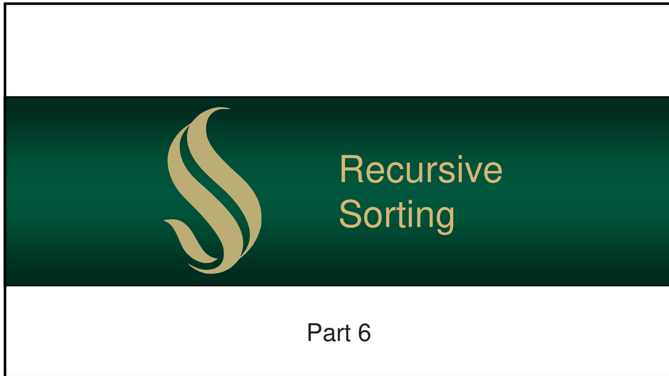

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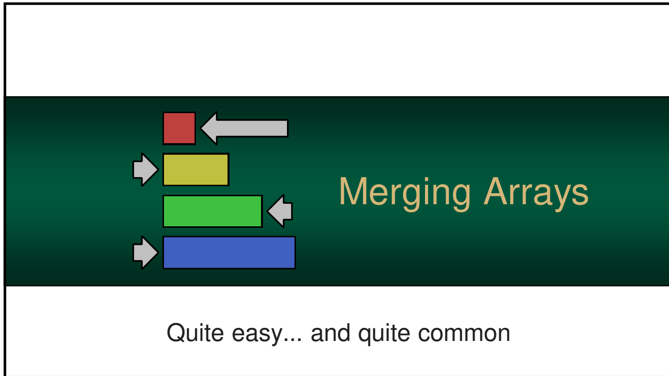

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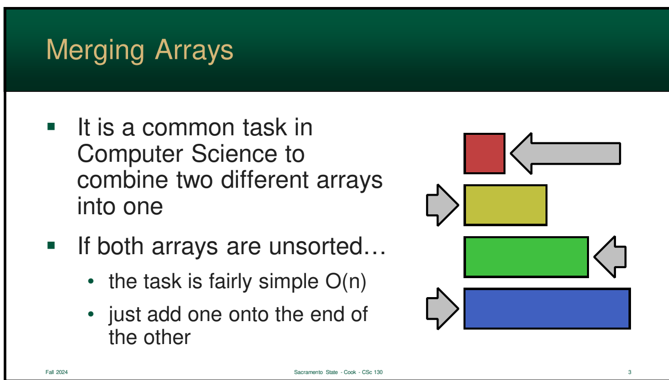

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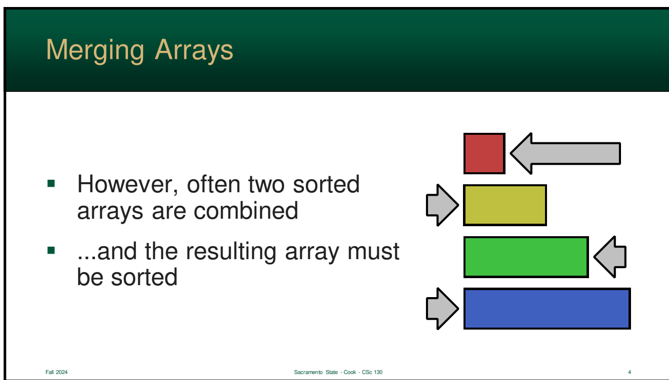

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Merging Arrays

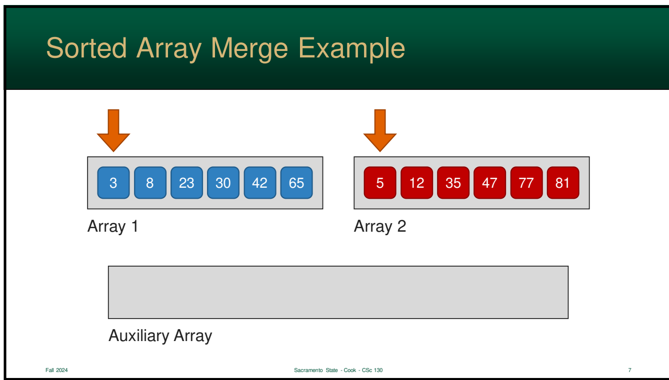
- The algorithm for merging two sorted arrays is very simple
- The resulting time complexity is O(n)
- However, it requires auxiliary storage of O(n)

5

Merge Algorithm

- Keep two counters – one for each array
- Loop while both arrays have data
  - take the smaller element and put it in the auxiliary array
  - increment the array's counter (which just lost an element)
- After the loop
  - one array will still have elements
  - append them to the auxiliary array

6



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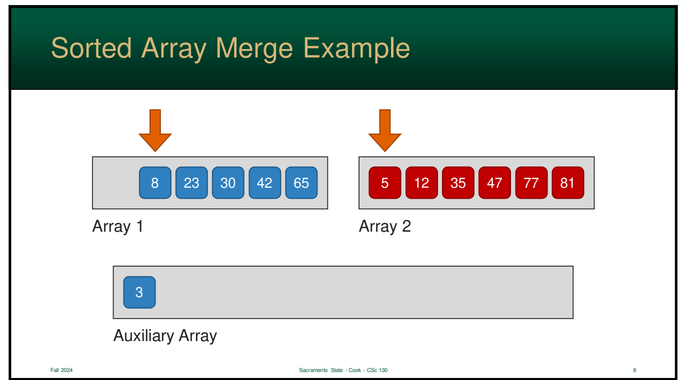

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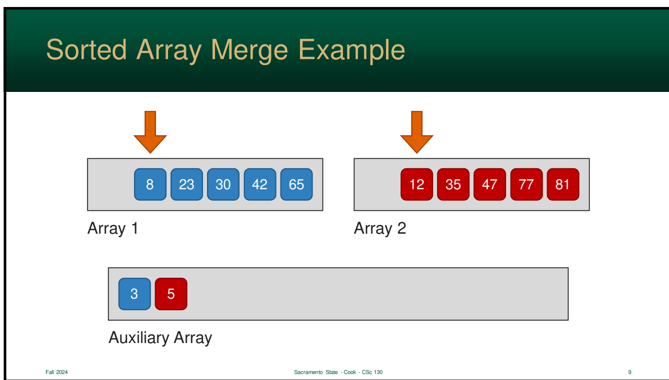

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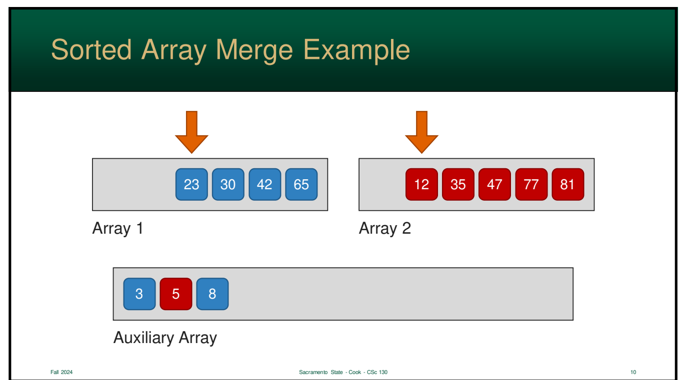

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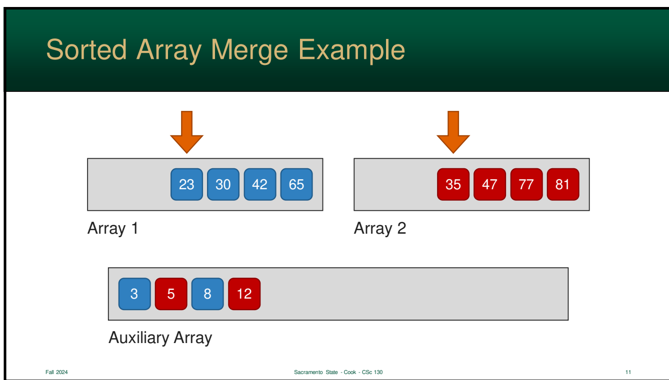

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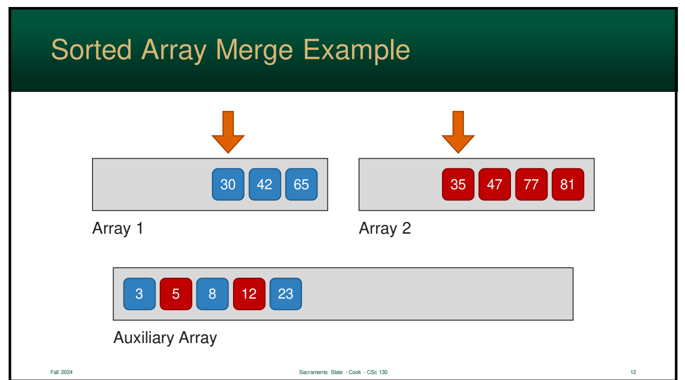

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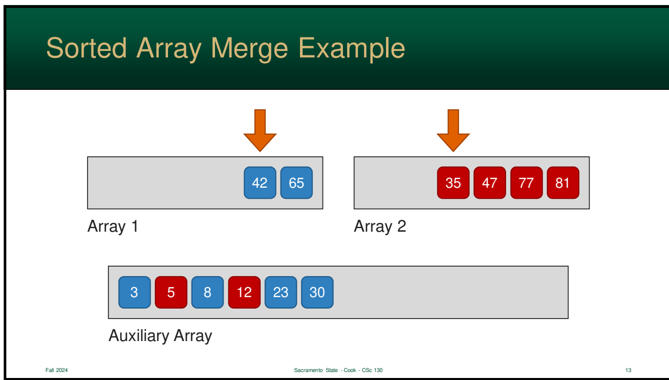

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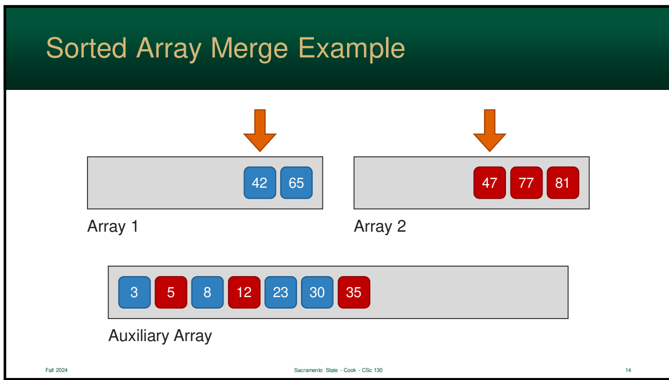

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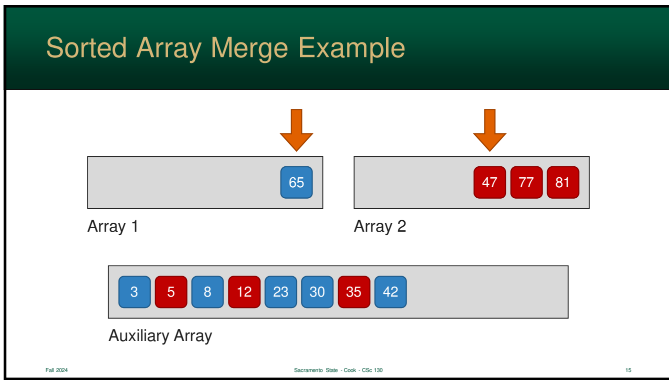

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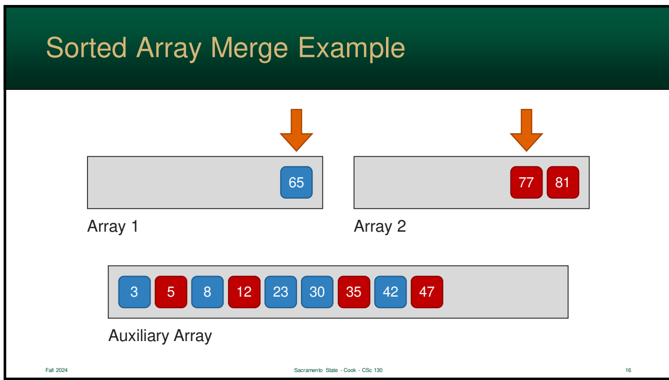

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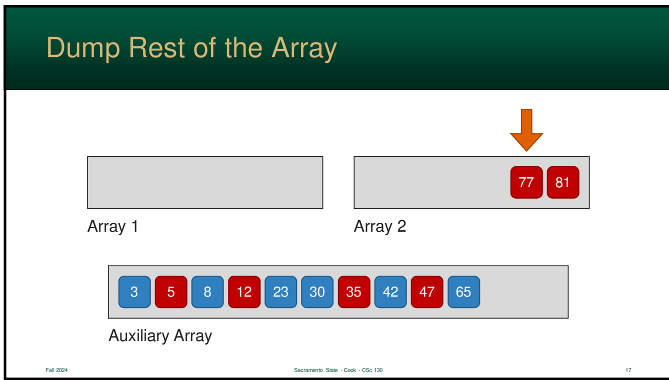

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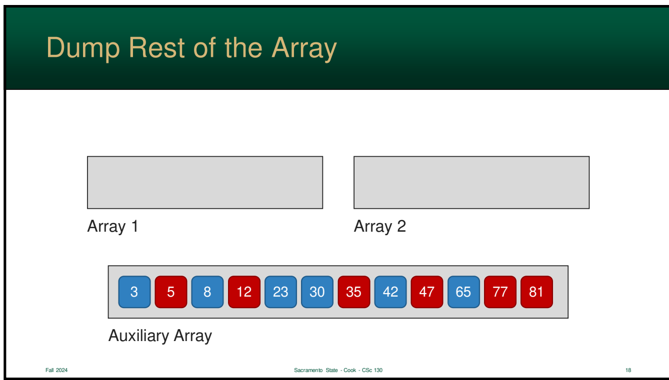

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Merge Sort

Divide and conquer!

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Merge Sort

- *Merge Sort* is a divide-and-conquer algorithm that cuts an array into smaller and smaller sublists until sorting them is arbitrary
- Invented by *John von Neumann* in 1945 *(19 BBW)*

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Merge Sort

- Because Merge-Sort defines a dividing the list into a list into smaller instances of itself, it naturally is solved using recursion
- Each recursive step cuts the list into two sublists until…
  - the list has 2 elements – arbitrary swap
  - the list has 1 element – which is, well, sorted

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Merge Sort

- As the recursion bubbles up, each sub list is merged using the algorithm we just discussed
- Divide-and-conquer algorithms ultimately result in O(n log n)
- Since an auxiliary array is required for the merge process, Merge-Sort, while fast, has O(n) auxiliary storage requirements

22

Merge Sort Example: Recurse down

62 44 4 80 13 53 35 16

62 44 4 80 | 13 53 35 16

62 44 | 4 80 | 13 53 | 35 16

23

Sort Merge Sort Example: Merge Up

44 62 | 4 80 | 13 53 | 16 35

4 44 62 80 | 13 16 35 53

4 13 16 35 44 53 62 80

24

Merge Sort Summary

Merge Sort	
Time Average	O(n log n)
Time Best	O(n log n)
Time Worst	O(n log n)
Auxiliary space	**O(n)**
Stable	Yes – Equal element order preserved
Online?	Yes – New data → new sublist

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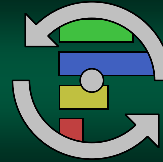


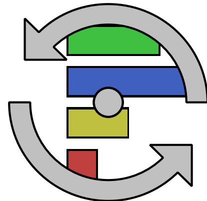
Quick Sort

Oh, I am getting dizzy....

26

Quick Sort

- *Quick-Sort* is a divide-and-conquer algorithm that rotates values around a *pivot*
- Invented by *C. A. R. Hoare* in 1959 *(5 BBW)*
- Even faster than both Merge Sort and Heap Sort
- ... but has a weaknesses



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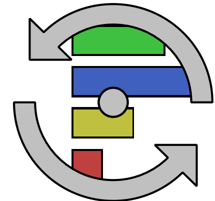
How it Works

- Like Merge-Sort, the array is broken down into smaller and smaller sub-lists
- However, before recursion
  - a value *p* is chosen in the sub-list as the *pivot* value
  - smaller items are moved before it
  - larger items are moved after it



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Choosing a Pivot

- Pivot can be any element in the sub-array
- …we need one actual value to compare
- This *pivot* is used to *partition* the values
- Different versions use different pivots
  - first item in the sub-array
  - end item in the sub-array
  - the midpoint of the sub-array
  - random value in the sub-array

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Partitioning the Values

- After the pivot *p* is selected, all elements are moved
- Two, separate, loops move through the elements and swaps elements less than/greater than the pivot
- The result is...
  - sub-array **L** contains items less than *p*
  - sub-array **G** contains items greater than *p*

30

Partitioning (pivot is the first item)

p p

L G

Partitioning the Values

- Note: neither **L** or **G** is sorted yet
- These will be called recursively by Quick-Sort
- Moving the elements, in-place, can look a tad ugly code-wise, but the logic is straight forward

p p

Partition Algorithm

- The sub-lists are stored in the original array – so there's no auxiliary storage
- The algorithm maintains two pointers
  - first moves left to right and keeps track of the values that are **too big**
  - second moves right to left and keeps track of the values that are **too small**
- Each moves independently

Partition Algorithm

- First move the **Too Big** pointer until a value is found that is bigger than the pivot
- Then move the **Too Small** pointer until a value is found that is smaller than Pivot
- Then, these values are swapped
- When the two pointers collide, we are done

Example Partition

- *In this example*, we pivot at the start of the array
- Any value can be used...
  - but it will have to be swapped to the start before the algorithm runs
  - this "saves" the pivot for later

42	8	12	77	65	30	47	52	9	35	23

Quick Sort Algorithm

```
while (tooBig < tooSmall)
{
   while (array[tooBig] <= array[pivot])
   {
       tooBig ++;
   }

   while (array[tooSmall] > array[pivot])
   {
       tooSmall --;
   }

   if (tooBig < tooSmall)
   {
      //swap array[tooBig] and array[tooSmall]
   }
}
//swap array[tooSmall] and array[pivot]
//Recurse QuickSort on both L and G
```

Example: Pivot is First

Too Big

Pivot

42	8	12	77	65	30	47	52	9	35	23

Too Small

37

Example: Move Too Big

Too Big

42	8	12	77	65	30	47	52	9	35	23

Too Small

38

Example: Move Too Big

Too Big

42	8	12	77	65	30	47	52	9	35	23

Too Small

39

Example: Move Too Big

Found!

Too Big

42	8	12	77	65	30	47	52	9	35	23

Too Small

40

Example: Now, Move Too Small

Too Big

42	8	12	77	65	30	47	52	9	35	23

Too Small

41

Example: Found (Immediately)

Too Big

42	8	12	77	65	30	47	52	9	35	23

Too Small

Found!

42

Example: Swap Values

Too Big

42 8 12 77 65 30 47 52 9 35 23

Too Small

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Example: Keep going… Move Too Big

Too Big

42 8 12 23 65 30 47 52 9 35 77

Too Small

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44

Example: Too Big Found

Found!

Too Big

42 8 12 23 65 30 47 52 9 35 77

Too Small

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45

Example: Move Too Small

Found!

Too Big

42 8 12 23 65 30 47 52 9 35 77

Too Small

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46

Example: Too Small Found

Too Big

42 8 12 23 65 30 47 52 9 35 77

Found!

Too Small

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47

Example: Swap Values

Too Big

42 8 12 23 65 30 47 52 9 35 77

Too Small

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Example: Keep going… Move Too Big

Too Big

42	8	12	23	35	30	47	52	9	65	77

Too Small

49

Example: Keep going… Move Too Big

Too Big

42	8	12	23	35	30	47	52	9	65	77

Too Small

50

Example: Too Big Found

Found!

Too Big

42	8	12	23	35	30	47	52	9	65	77

Too Small

51

Example: Move Too Small

Too Big

42	8	12	23	35	30	47	52	9	65	77

Too Small

52

Example: Too Small Found

Too Big

42	8	12	23	35	30	47	52	9	65	77

Found!

Too Small

53

Example: Swap Values

Too Big

42	8	12	23	35	30	47	52	9	65	77

Too Small

54

Example: Keep going… Move Too Big

Too Big

42	8	12	23	35	30	9	52	47	65	77

Too Small

55

Example: Too Big Found

Found!

Too Big

42	8	12	23	35	30	9	52	47	65	77

Too Small

56

Example: Move Too Small

Too Big

42	8	12	23	35	30	9	52	47	65	77

Too Small

57

Example: Move Too Small

Too Big

42	8	12	23	35	30	9	52	47	65	77

Too Small

58

Example: Pointers Passed Each Other

Too Big

42	8	12	23	35	30	9	52	47	65	77

Too Small

59

Example: Swap Pivot & Too Small

Too Big

42	8	12	23	35	30	9	52	47	65	77

Too Small

60

Example: Done (with this pass)

9 | 8 | 12 | 23 | 35 | 30 | 42 | 52 | 47 | 65 | 77

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Recursion Time!

- Notice: all the items before the pivot are smaller and all the items after are a larger
- Now, we can recurse both sides
- The result is a sorted array

[9 8 12 23 35 30] [42] [52 47 65 77]

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Quick Sort Example

[44 62 4 80 13 53 35 16 77]

[16 35 13 4] [44] [77 53 80 62]

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63

Quick Sort Example

[44 62 4 80 13 53 35 16 77]

[16 35 13 4] [44] [77 53 80 62]

[4 13] [16] [35] [62 53] [77] [80]

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Quick Sort Example

Sorted on base case

[4 13] [16] [35] [44] [53 62] [77] [80]

In the main array from the first partition

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Quick Sort: Worst Case

- Assume we get array that is already sorted
- This can cause huge problems!
- Shockingly, the efficiently of this sort can degenerate if we are not careful

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Quick Sort: Worst Case

- If the first item is the pivot
  - a sorted array will cause both the pointers will pass simply pass each other
  - one sub-array will be empty, the second will contains **ALL** the elements – 1
- If the last item is the pivot
  - reverse sorted array will have the same effect

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Quick Sort: Worst Case

Pivot
Too Big
4 13 16 35 44 53 62 77 80 82 92
Too Small

68

Worst Case: Move Too Big

Too Big
Found immediately
4 13 16 35 44 53 62 77 80 82 92
Too Small

69

Worst Case: Now, Move Too Small

Too Big
4 13 16 35 44 53 62 77 80 82 92
Too Small

70

Worst Case: Pointers Passed

Too Big
4 13 16 35 44 53 62 77 80 82 92
Too Small

71

Worst Case: Recurse on n-1

Uh, oh! We will now recurse on all n - 1 items!
4 13 16 35 44 53 62 77 80 82 92

72

Quick Sort Analysis

- So, in the worst case, Quick Sort is **$O(n^2)$**
- ... and, given all the work it has to do with the pointers, it gets beat by Bubble Sort

73

How Can We Avoid This?

- If you don't know if the array is randomized, *manually randomize the values*
- O(n) – run i from first to last element and swap array[i] and array [random]

74

Quick Sort Summary

Quick Sort	
Time Average	O(n log n)
Time Best	O(n log n)
Time Worst	**$O(n^2)$**
Auxiliary space	O(1)
Stable	No – Equal element order not preserved
Online?	No

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