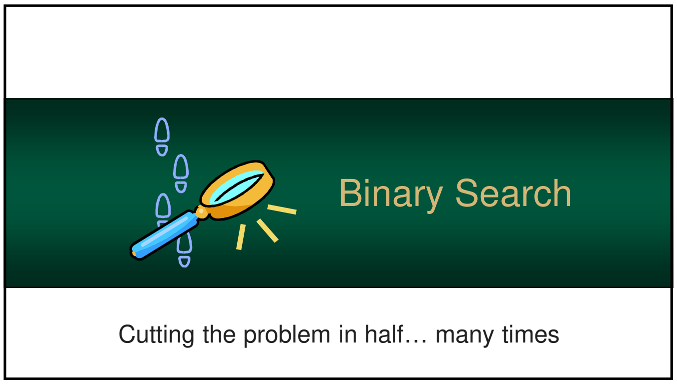
Binary Search & Sorting

Part 5

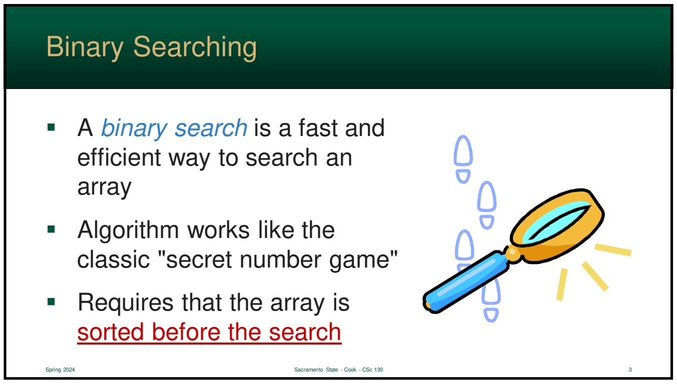
1



Binary Search

Cutting the problem in half… many times

2



Binary Searching

- A *binary search* is a fast and efficient way to search an array
- Algorithm works like the classic "secret number game"
- Requires that the array is sorted before the search

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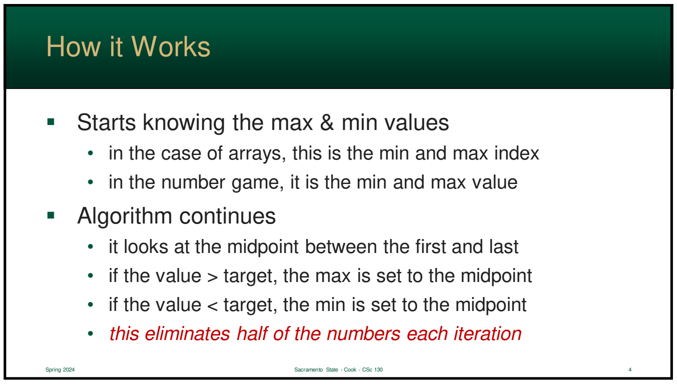

How it Works

- Starts knowing the max & min values
  - in the case of arrays, this is the min and max index
  - in the number game, it is the min and max value
- Algorithm continues
  - it looks at the midpoint between the first and last
  - if the value > target, the max is set to the midpoint
  - if the value < target, the min is set to the midpoint
  - *this eliminates half of the numbers each iteration*

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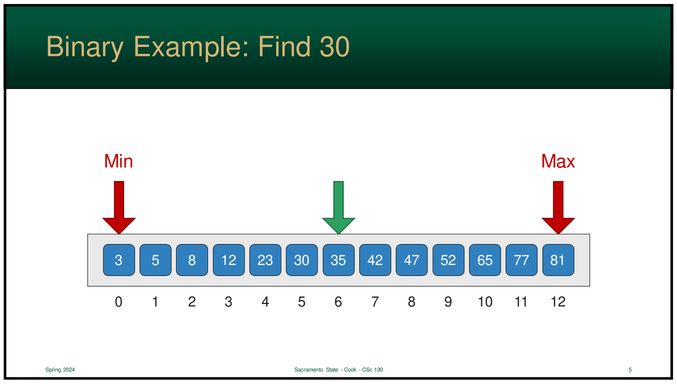

Binary Example: Find 30

3	5	8	12	23	30	35	42	47	52	65	77	81
0	1	2	3	4	5	6	7	8	9	10	11	12

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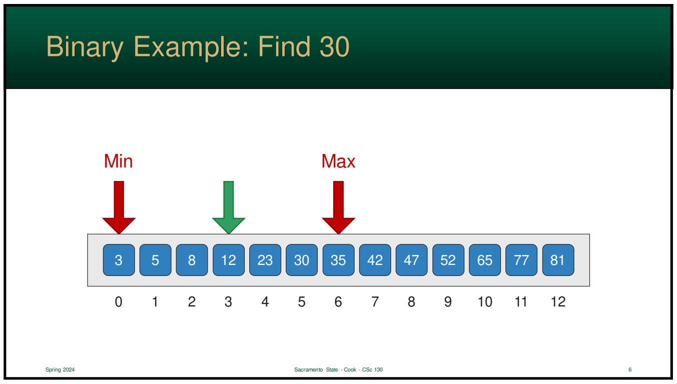

Binary Example: Find 30

3	5	8	12	23	30	35	42	47	52	65	77	81
0	1	2	3	4	5	6	7	8	9	10	11	12

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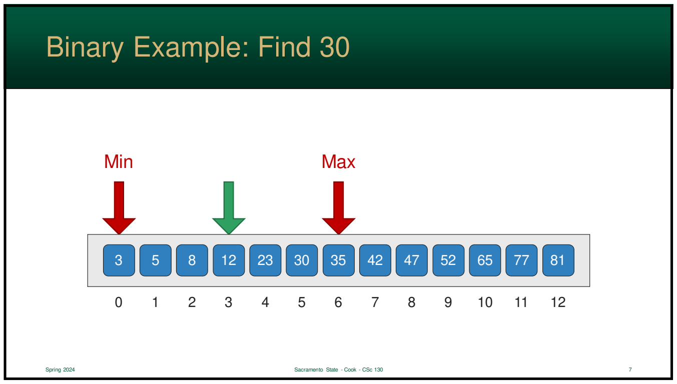

Binary Example: Find 30

0	1	2	3	4	5	6	7	8	9	10	11	12
3	5	8	12	23	30	35	42	47	52	65	77	81

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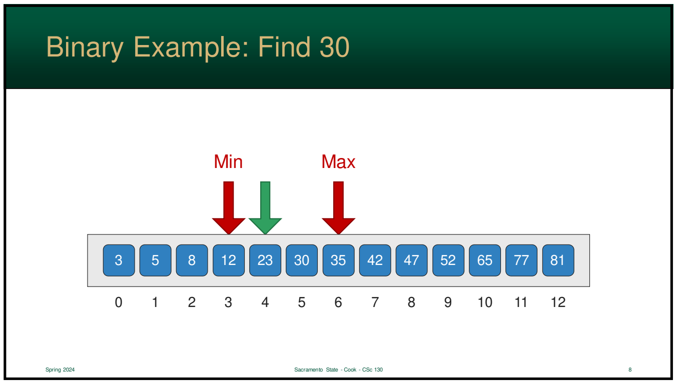

Binary Example: Find 30

0	1	2	3	4	5	6	7	8	9	10	11	12
3	5	8	12	23	30	35	42	47	52	65	77	81

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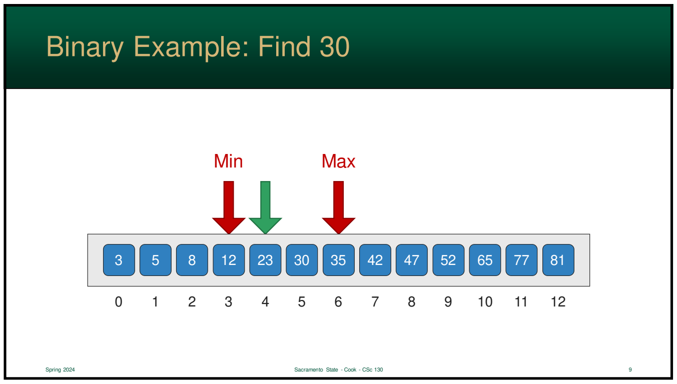

Binary Example: Find 30

0	1	2	3	4	5	6	7	8	9	10	11	12
3	5	8	12	23	30	35	42	47	52	65	77	81

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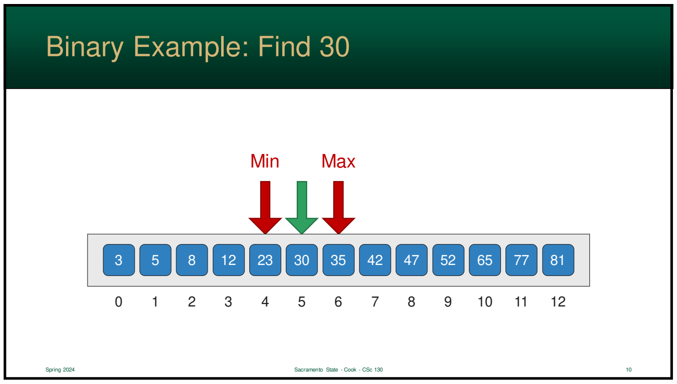

Binary Example: Find 30

0	1	2	3	4	5	6	7	8	9	10	11	12
3	5	8	12	23	30	35	42	47	52	65	77	81

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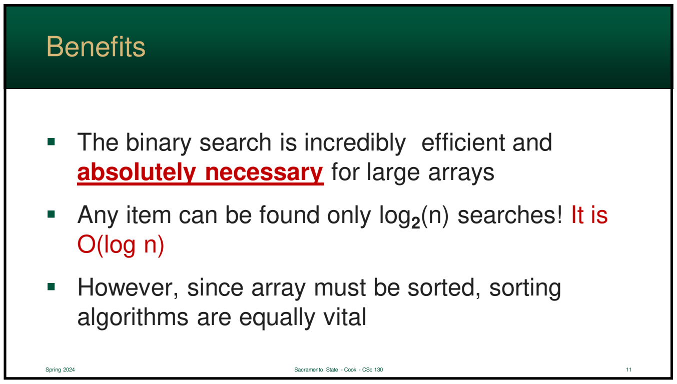

Benefits

- The binary search is incredibly efficient and **absolutely necessary** for large arrays
- Any item can be found only $\log_2(n)$ searches! It is O(log n)
- However, since array must be sorted, sorting algorithms are equally vital

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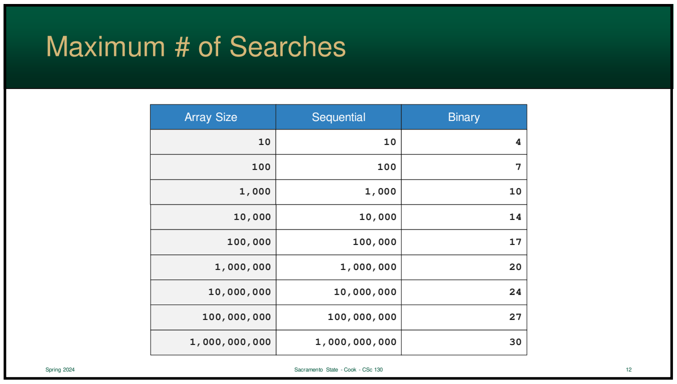
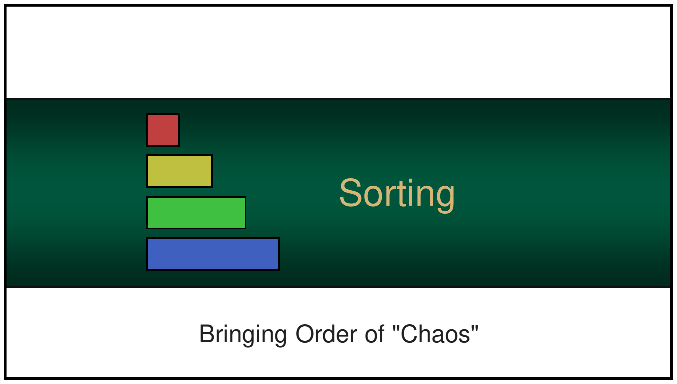


Maximum # of Searches

Array Size	Sequential	Binary
10	10	4
100	100	7
1,000	1,000	10
10,000	10,000	14
100,000	100,000	17
1,000,000	1,000,000	20
10,000,000	10,000,000	24
100,000,000	100,000,000	27
1,000,000,000	1,000,000,000	30

12



Sorting

Bringing Order of "Chaos"

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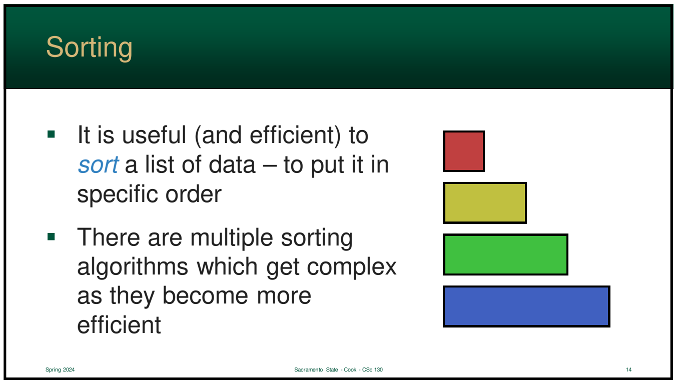

Sorting

- It is useful (and efficient) to *sort* a list of data – to put it in specific order
- There are multiple sorting algorithms which get complex as they become more efficient

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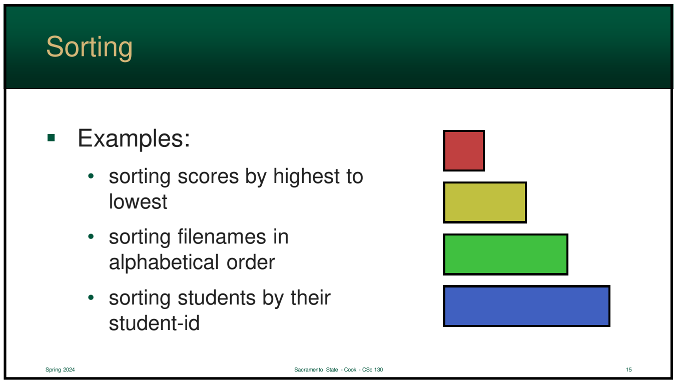

Sorting

- Examples:
  - sorting scores by highest to lowest
  - sorting filenames in alphabetical order
  - sorting students by their student-id

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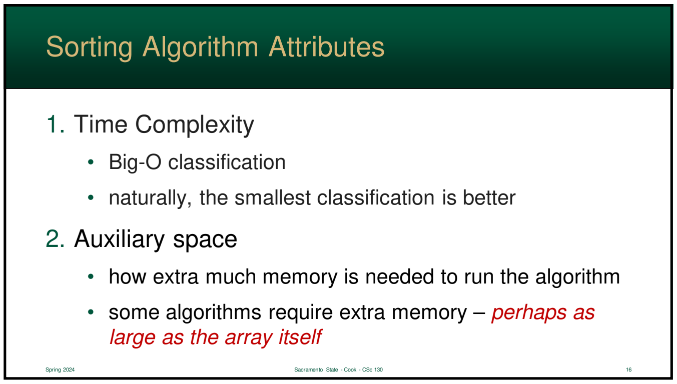

Sorting Algorithm Attributes

1. Time Complexity
   - Big-O classification
   - naturally, the smallest classification is better
2. Auxiliary space
   - how extra much memory is needed to run the algorithm
   - some algorithms require extra memory – *perhaps as large as the array itself*

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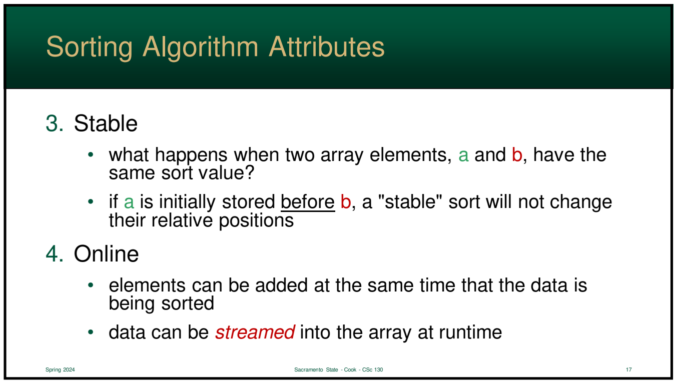

Sorting Algorithm Attributes

3. Stable
   - what happens when two array elements, a and b, have the same sort value?
   - if a is initially stored before b, a "stable" sort will not change their relative positions
4. Online
   - elements can be added at the same time that the data is being sorted
   - data can be *streamed* into the array at runtime

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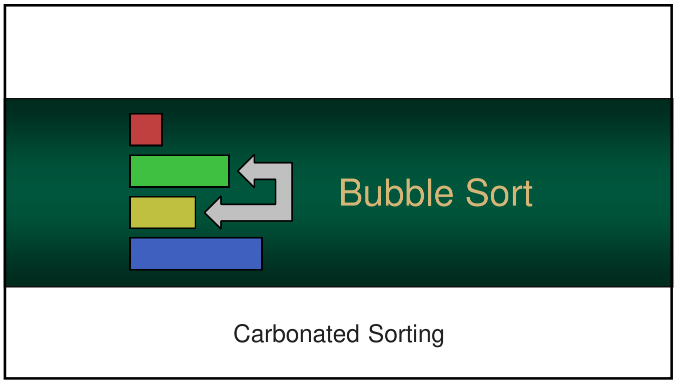

Bubble Sort

Carbonated Sorting

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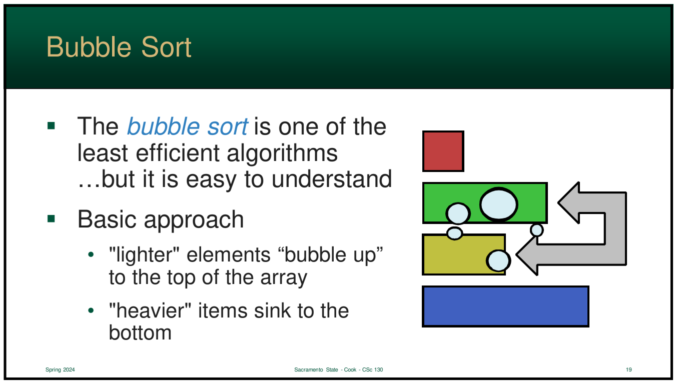

Bubble Sort

- The *bubble sort* is one of the least efficient algorithms …but it is easy to understand
- Basic approach
  - "lighter" elements “bubble up” to the top of the array
  - "heavier" items sink to the bottom

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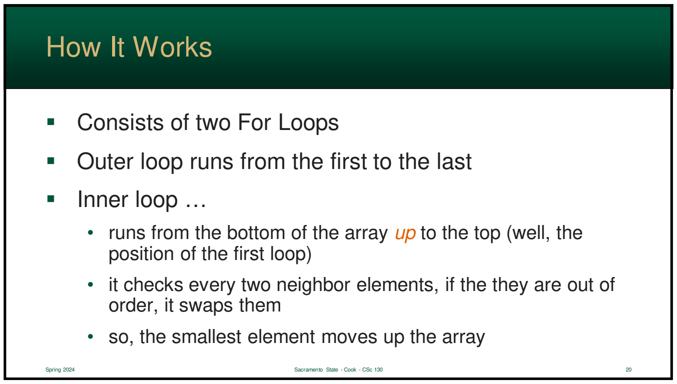

How It Works

- Consists of two For Loops
- Outer loop runs from the first to the last
- Inner loop …
  - runs from the bottom of the array *up* to the top (well, the position of the first loop)
  - it checks every two neighbor elements, if the they are out of order, it swaps them
  - so, the smallest element moves up the array

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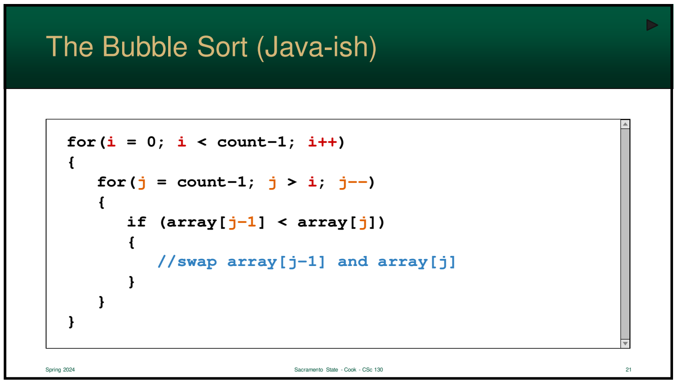

The Bubble Sort (Java-ish)

```
for(i = 0; i < count-1; i++)
{
   for(j = count-1; j > i; j--)
   {
      if (array[j-1] < array[j])
      {
         //swap array[j-1] and array[j]
      }
   }
}
```

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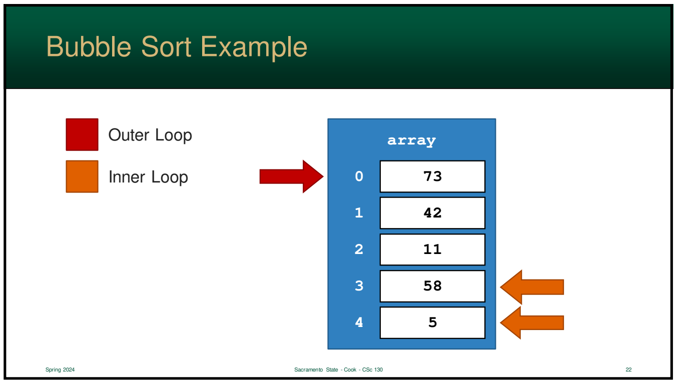


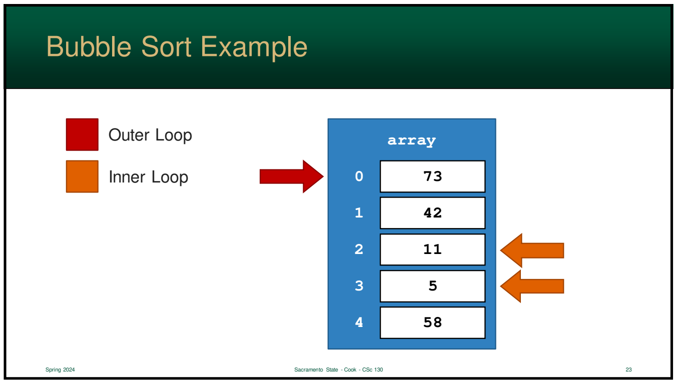
Bubble Sort Example

🟥 Outer Loop

🟧 Inner Loop

	array
0	73
1	42
2	11
3	58
4	5

22



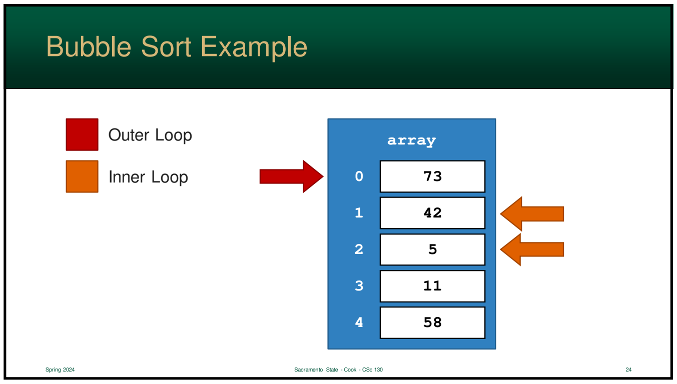
Bubble Sort Example

🟥 Outer Loop

🟧 Inner Loop

	array
0	73
1	42
2	11
3	5
4	58

23



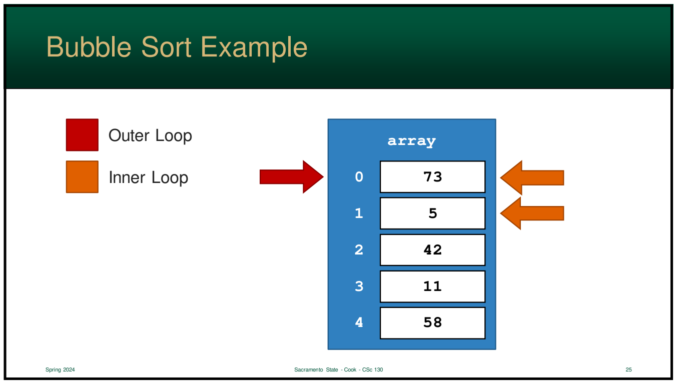
Bubble Sort Example

🟥 Outer Loop

🟧 Inner Loop

	array
0	73
1	42
2	5
3	11
4	58

24



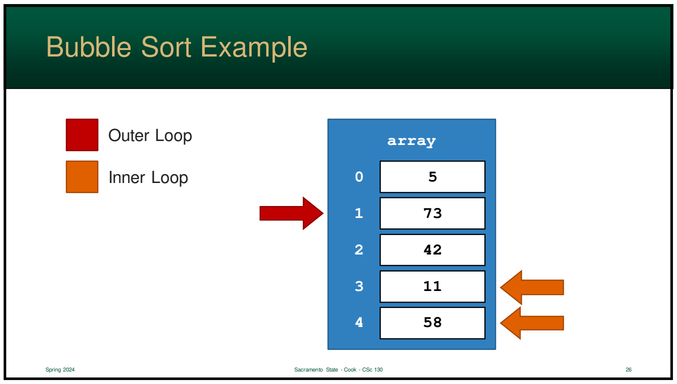
Bubble Sort Example

🟥 Outer Loop

🟧 Inner Loop

		array	
➡	0	73	⬅
	1	5	⬅
	2	42	
	3	11	
	4	58	

25



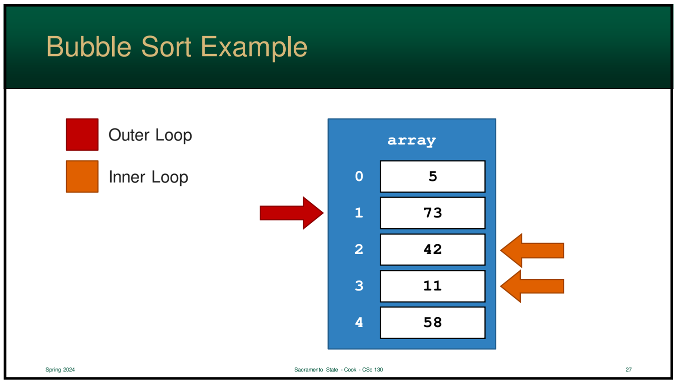
Bubble Sort Example

🟥 Outer Loop

🟧 Inner Loop

		array	
	0	5	
➡	1	73	
	2	42	
	3	11	⬅
	4	58	⬅

26



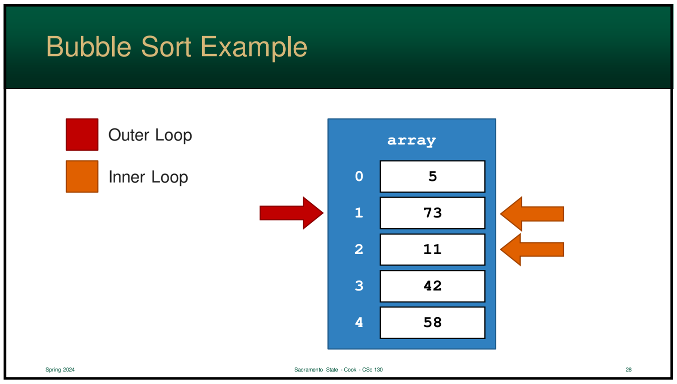
Bubble Sort Example

🟥 Outer Loop

🟧 Inner Loop

		array	
	0	5	
➡	1	73	
	2	42	⬅
	3	11	⬅
	4	58	

27



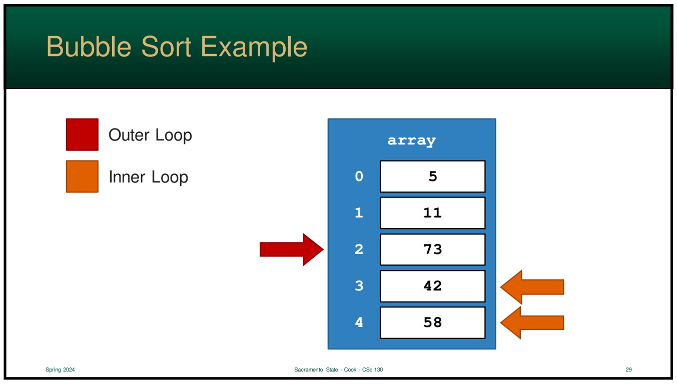
Bubble Sort Example

🟥 Outer Loop

🟧 Inner Loop

		array	
	0	5	
➡	1	73	⬅
	2	11	⬅
	3	42	
	4	58	

28



Bubble Sort Example

🟥 Outer Loop

🟧 Inner Loop

		array	
	0	5	
	1	11	
➡	2	73	
	3	42	⬅
	4	58	⬅

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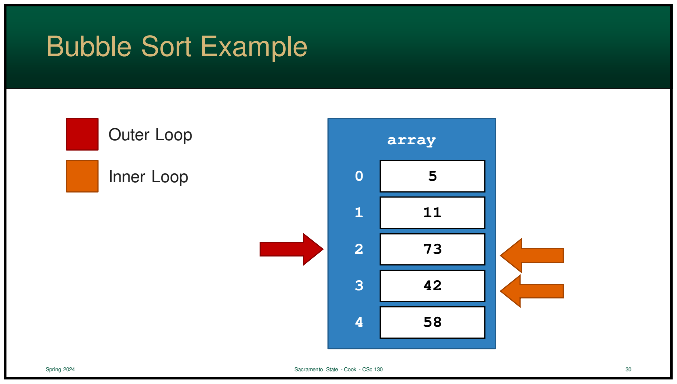


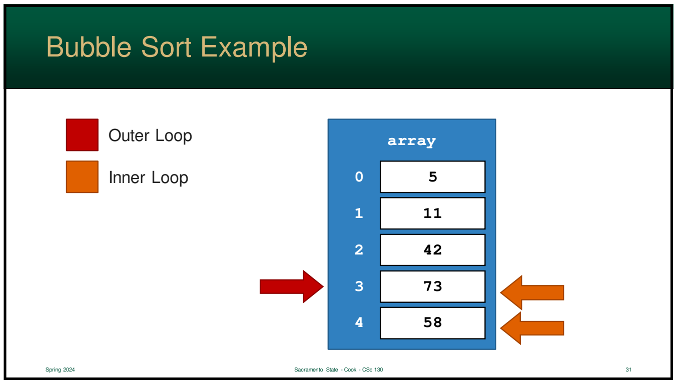
Bubble Sort Example

🟥 Outer Loop

🟧 Inner Loop

		array	
	0	5	
	1	11	
➡	2	73	⬅
	3	42	⬅
	4	58	

30



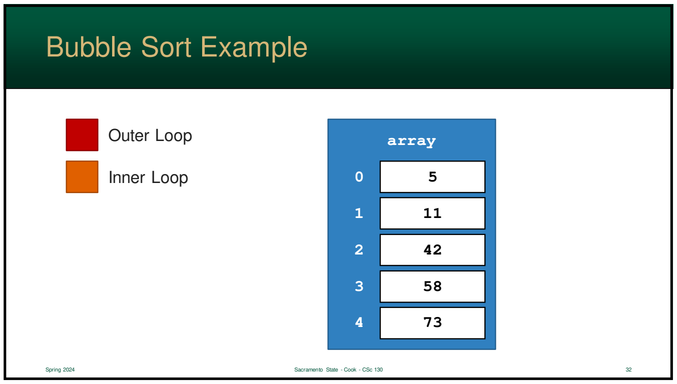
Bubble Sort Example

🟥 Outer Loop

🟧 Inner Loop

array	
0	5
1	11
2	42
3	73
4	58

31



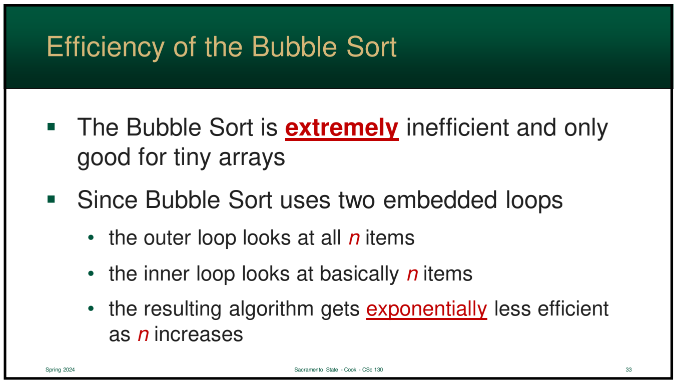
Bubble Sort Example

🟥 Outer Loop

🟧 Inner Loop

array	
0	5
1	11
2	42
3	58
4	73

32



Efficiency of the Bubble Sort

- The Bubble Sort is **extremely** inefficient and only good for tiny arrays
- Since Bubble Sort uses two embedded loops
  - the outer loop looks at all *n* items
  - the inner loop looks at basically *n* items
  - the resulting algorithm gets exponentially less efficient as *n* increases

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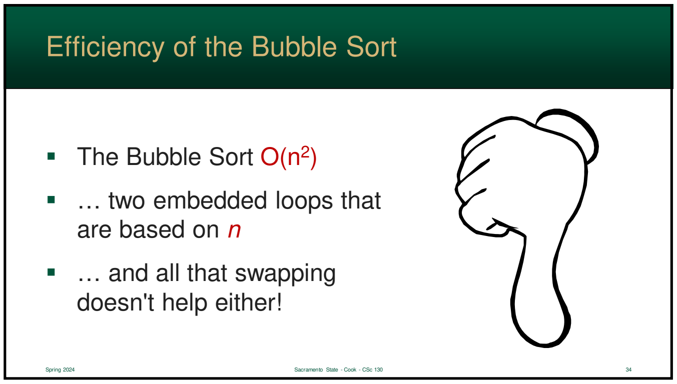

Efficiency of the Bubble Sort

- The Bubble Sort $O(n^2)$
- … two embedded loops that are based on *n*
- … and all that swapping doesn't help either!

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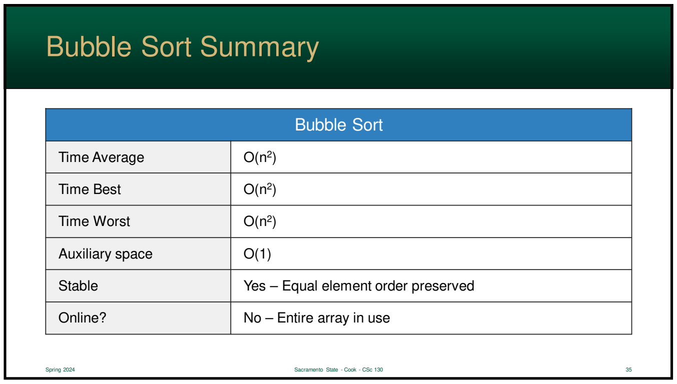

Bubble Sort Summary

Bubble Sort	
Time Average	$O(n^2)$
Time Best	$O(n^2)$
Time Worst	$O(n^2)$
Auxiliary space	O(1)
Stable	Yes – Equal element order preserved
Online?	No – Entire array in use

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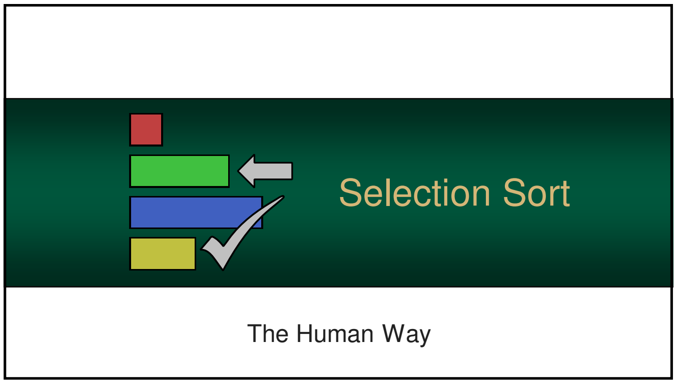

Selection Sort

The Human Way

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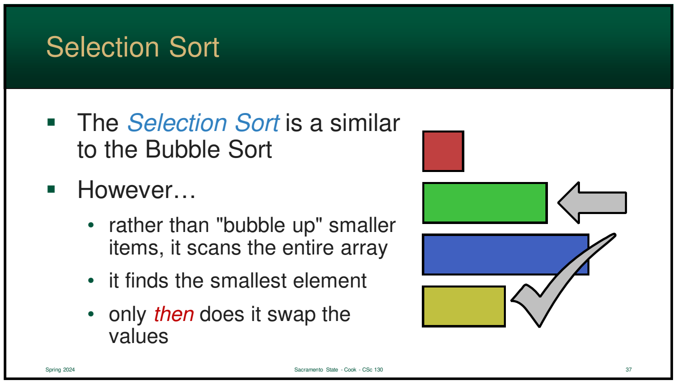

Selection Sort

- The *Selection Sort* is a similar to the Bubble Sort
- However…
  - rather than "bubble up" smaller items, it scans the entire array
  - it finds the smallest element
  - only *then* does it swap the values

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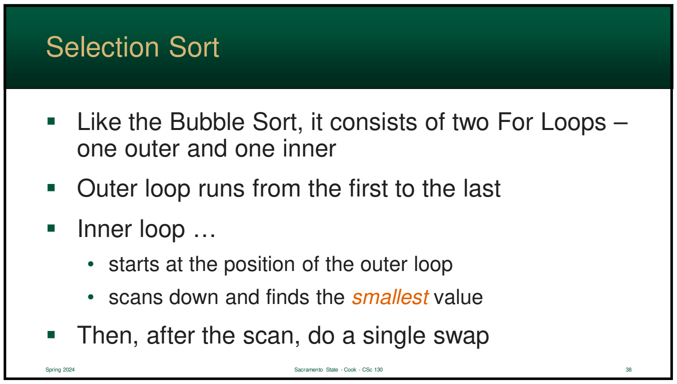

Selection Sort

- Like the Bubble Sort, it consists of two For Loops – one outer and one inner
- Outer loop runs from the first to the last
- Inner loop …
  - starts at the position of the outer loop
  - scans down and finds the *smallest* value
- Then, after the scan, do a single swap

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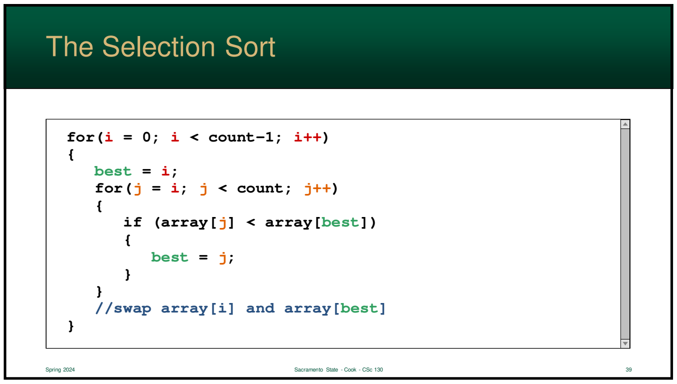

The Selection Sort

```
for(i = 0; i < count-1; i++)
{
   best = i;
   for(j = i; j < count; j++)
   {
      if (array[j] < array[best])
      {
         best = j;
      }
   }
   //swap array[i] and array[best]
}
```

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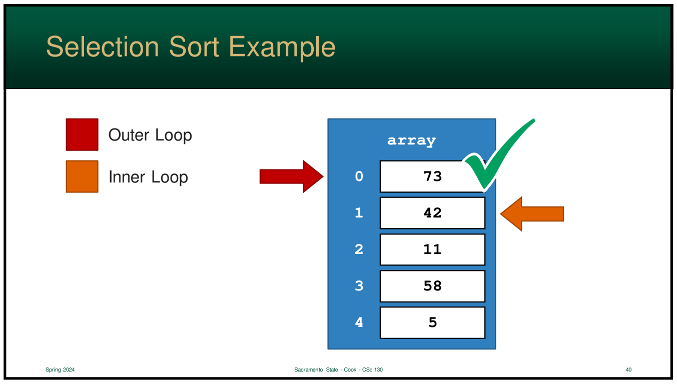


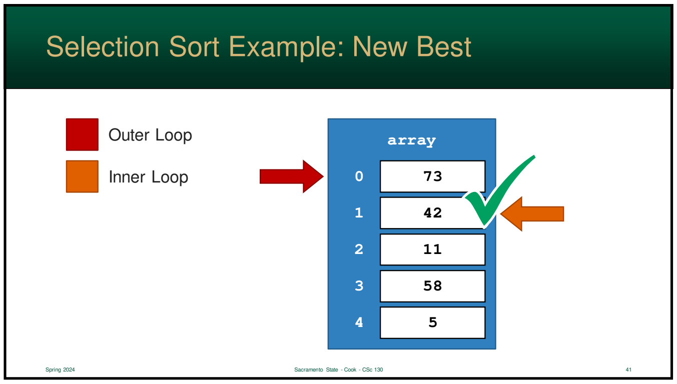
Selection Sort Example

🟥 Outer Loop

🟧 Inner Loop

	array
0	73
1	42
2	11
3	58
4	5

40



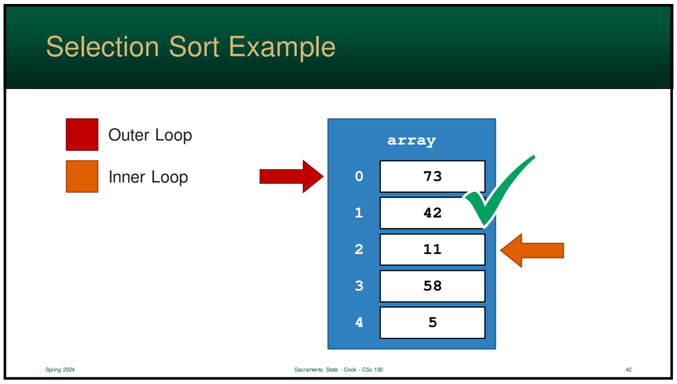
Selection Sort Example: New Best

🟥 Outer Loop

🟧 Inner Loop

	array
0	73
1	42
2	11
3	58
4	5

41



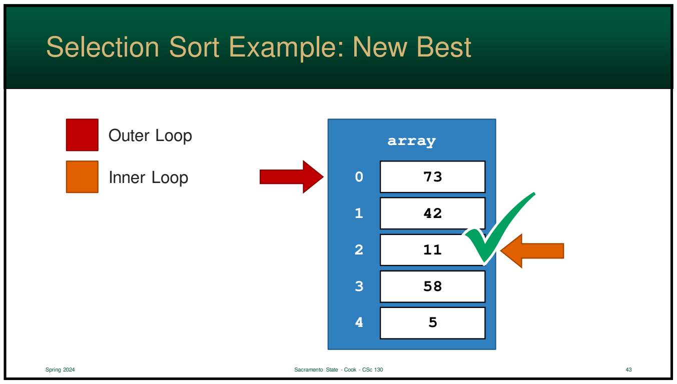
Selection Sort Example

🟥 Outer Loop

🟧 Inner Loop

	array
0	73
1	42
2	11
3	58
4	5

42



Selection Sort Example: New Best

🟥 Outer Loop

🟧 Inner Loop

	array
0	73
1	42
2	11
3	58
4	5

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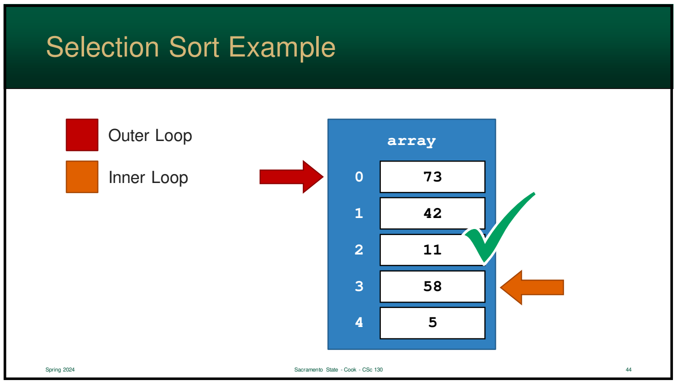

Selection Sort Example

🟥 Outer Loop

🟧 Inner Loop

	array
0	73
1	42
2	11
3	58
4	5

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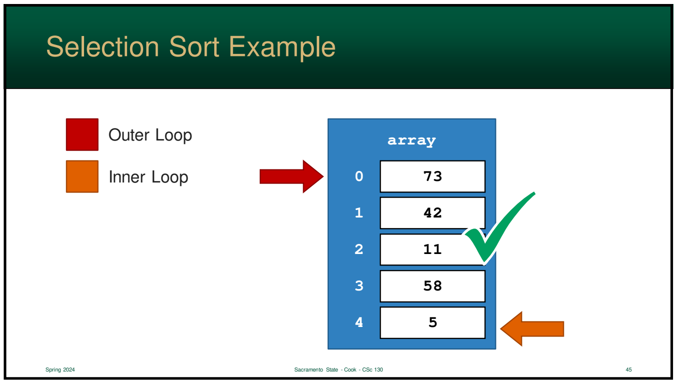

Selection Sort Example

🟥 Outer Loop

🟧 Inner Loop

	array
0	73
1	42
2	11
3	58
4	5

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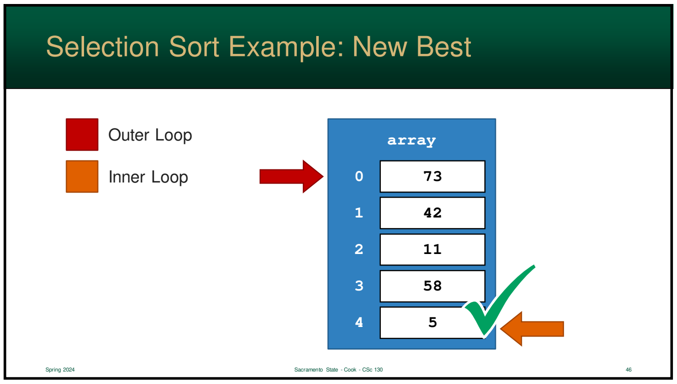

Selection Sort Example: New Best

🟥 Outer Loop

🟧 Inner Loop

	array
0	73
1	42
2	11
3	58
4	5

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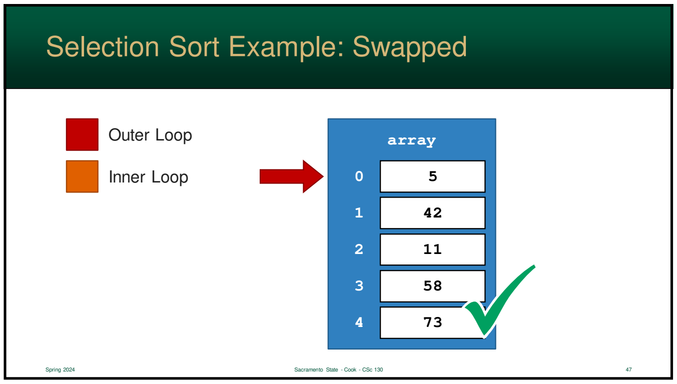

Selection Sort Example: Swapped

🟥 Outer Loop

🟧 Inner Loop

	array
0	5
1	42
2	11
3	58
4	73

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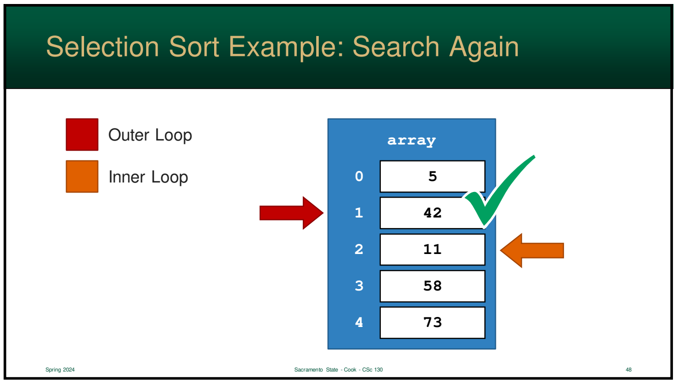

Selection Sort Example: Search Again

🟥 Outer Loop

🟧 Inner Loop

	array
0	5
1	42
2	11
3	58
4	73

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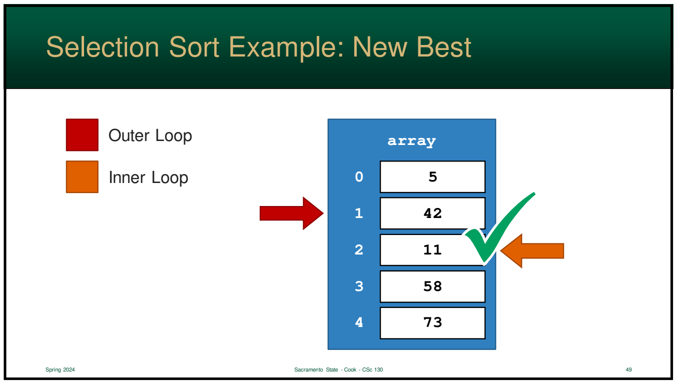

Selection Sort Example: New Best

🟥 Outer Loop

🟧 Inner Loop

	array			
	0	5		
→	1	42		
	2	11	✓	←
	3	58		
	4	73		

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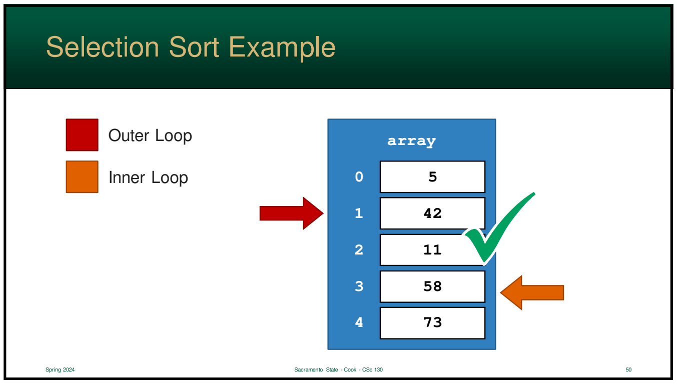

Selection Sort Example

🟥 Outer Loop

🟧 Inner Loop

	array			
	0	5		
→	1	42		
	2	11	✓	
	3	58		←
	4	73		

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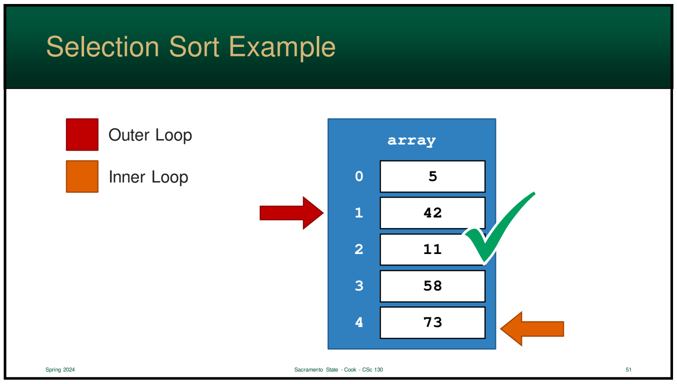
50



Selection Sort Example

🟥 Outer Loop

🟧 Inner Loop

	array			
	0	5		
→	1	42		
	2	11	✓	
	3	58		
	4	73		←

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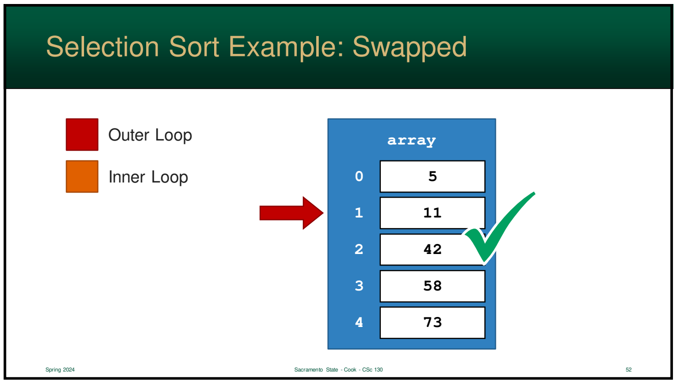
51



Selection Sort Example: Swapped

🟥 Outer Loop

🟧 Inner Loop

	array		
	0	5	
→	1	11	
	2	42	✓
	3	58	
	4	73	

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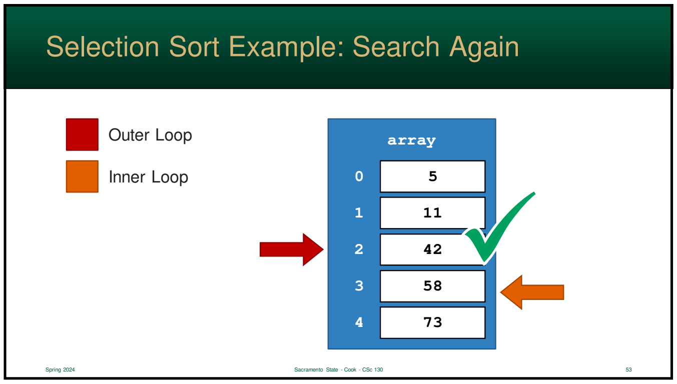
52



Selection Sort Example: Search Again

🟥 Outer Loop

🟧 Inner Loop

	array			
	0	5		
	1	11		
→	2	42	✓	
	3	58		←
	4	73		

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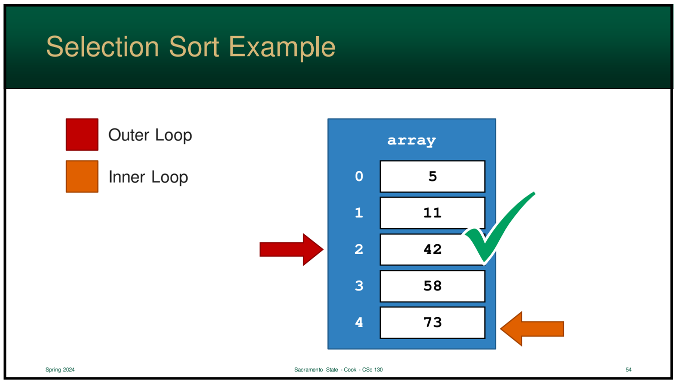
53



Selection Sort Example

🟥 Outer Loop

🟧 Inner Loop

	array			
	0	5		
	1	11		
→	2	42	✓	
	3	58		
	4	73		←

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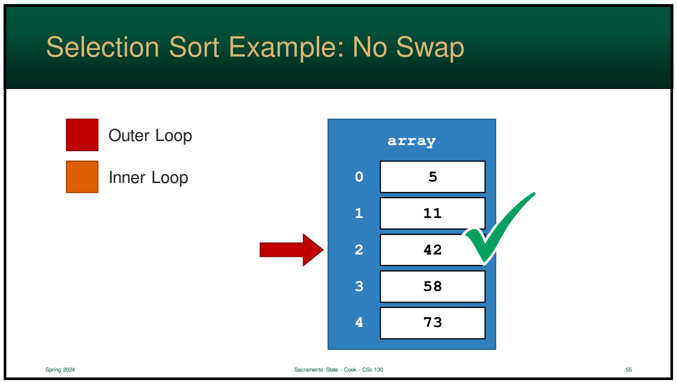
54



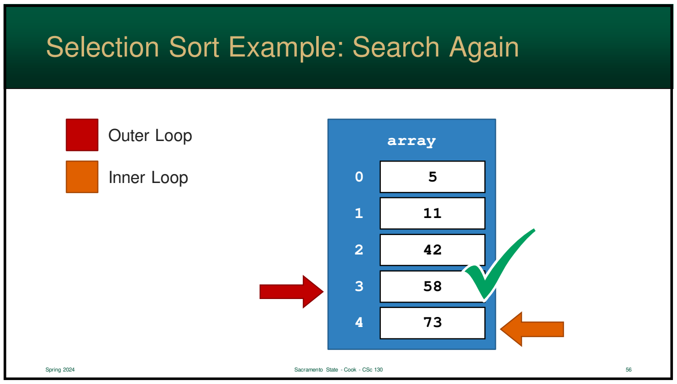
Selection Sort Example: No Swap

🟥 Outer Loop

🟧 Inner Loop

array	
0	5
1	11
2	42
3	58
4	73

55



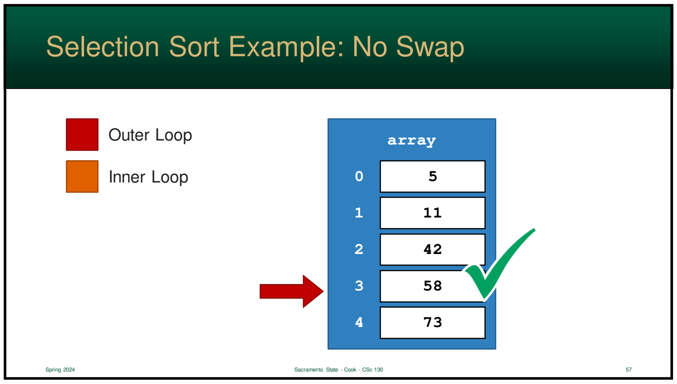
Selection Sort Example: Search Again

🟥 Outer Loop

🟧 Inner Loop

array	
0	5
1	11
2	42
3	58
4	73

56



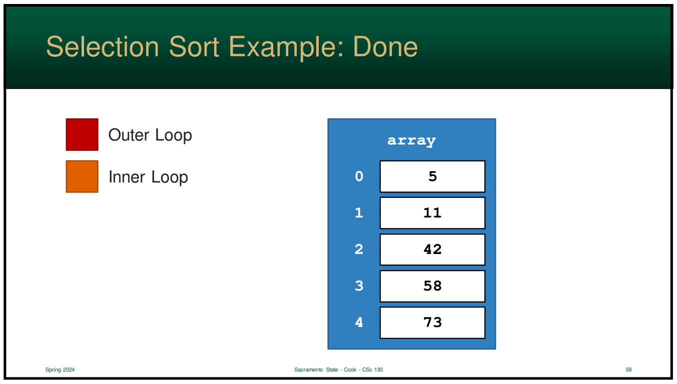
Selection Sort Example: No Swap

🟥 Outer Loop

🟧 Inner Loop

array	
0	5
1	11
2	42
3	58
4	73

57



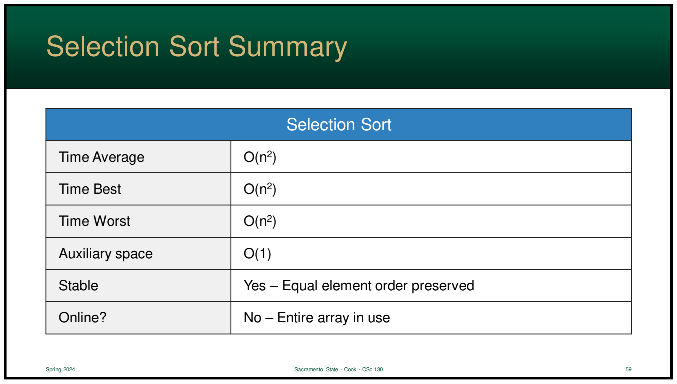
Selection Sort Example: Done

🟥 Outer Loop

🟧 Inner Loop

array	
0	5
1	11
2	42
3	58
4	73

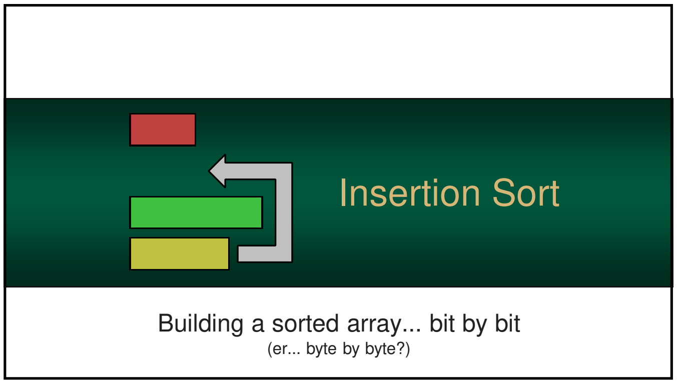
58



Selection Sort Summary

Selection Sort	
Time Average	O(n^2)
Time Best	O(n^2)
Time Worst	O(n^2)
Auxiliary space	O(1)
Stable	Yes – Equal element order preserved
Online?	No – Entire array in use

59

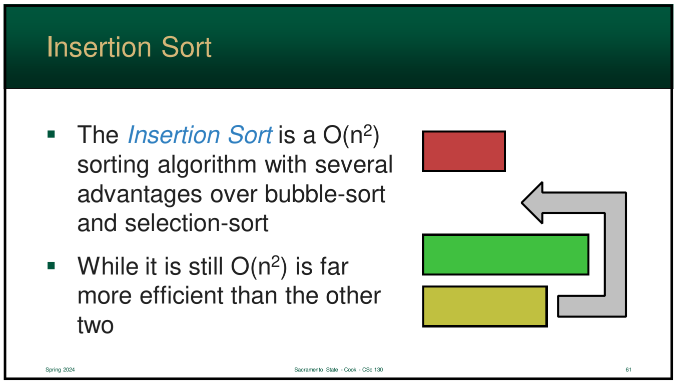


Insertion Sort

Building a sorted array... bit by bit

(er... byte by byte?)

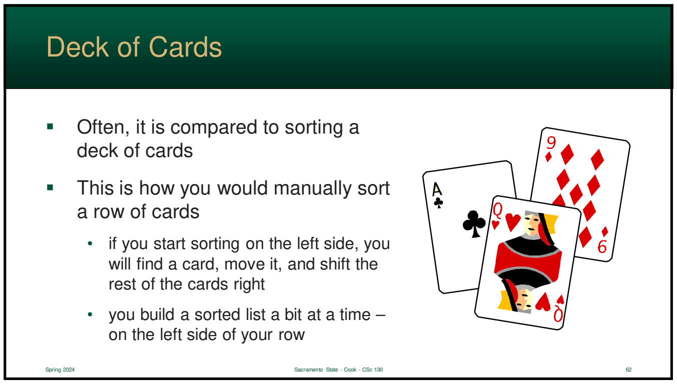
60



Insertion Sort

- The *Insertion Sort* is a O(n^2) sorting algorithm with several advantages over bubble-sort and selection-sort
- While it is still O(n^2) is far more efficient than the other two

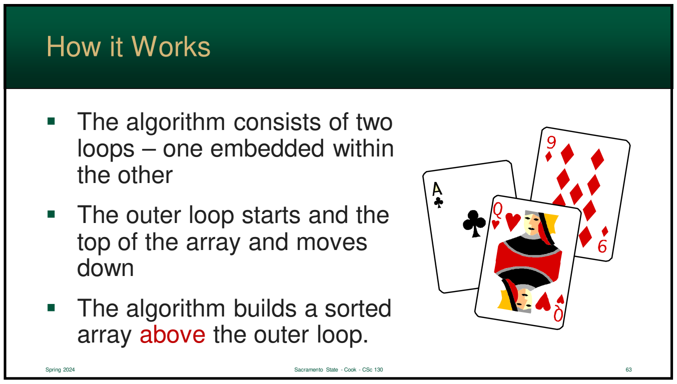
61



Deck of Cards

- Often, it is compared to sorting a deck of cards
- This is how you would manually sort a row of cards
  - if you start sorting on the left side, you will find a card, move it, and shift the rest of the cards right
  - you build a sorted list a bit at a time – on the left side of your row

62



How it Works

- The algorithm consists of two loops – one embedded within the other
- The outer loop starts and the top of the array and moves down
- The algorithm builds a sorted array above the outer loop.

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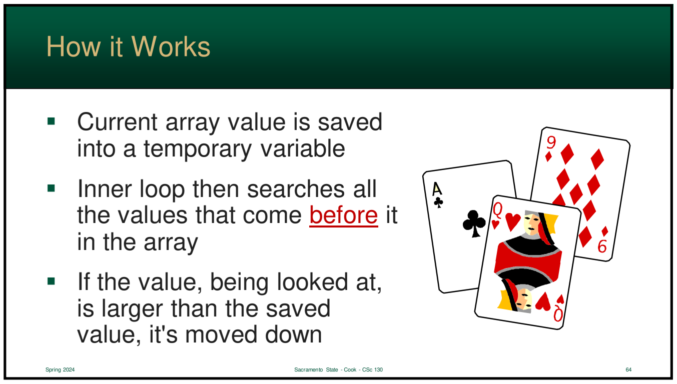

How it Works

- Current array value is saved into a temporary variable
- Inner loop then searches all the values that come before it in the array
- If the value, being looked at, is larger than the saved value, it's moved down

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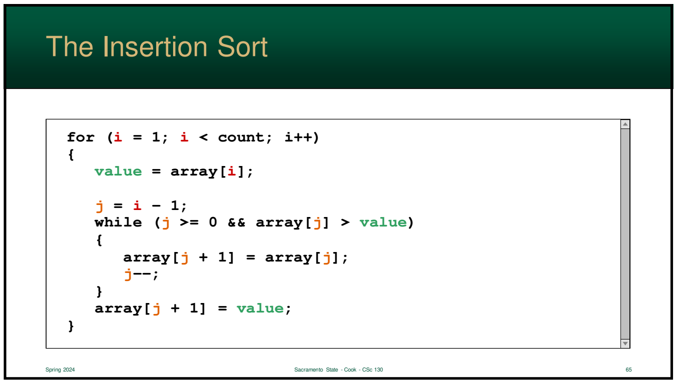

The Insertion Sort

```
for (i = 1; i < count; i++)
{
   value = array[i];

   j = i - 1;
   while (j >= 0 && array[j] > value)
   {
      array[j + 1] = array[j];
      j--;
   }
   array[j + 1] = value;
}
```

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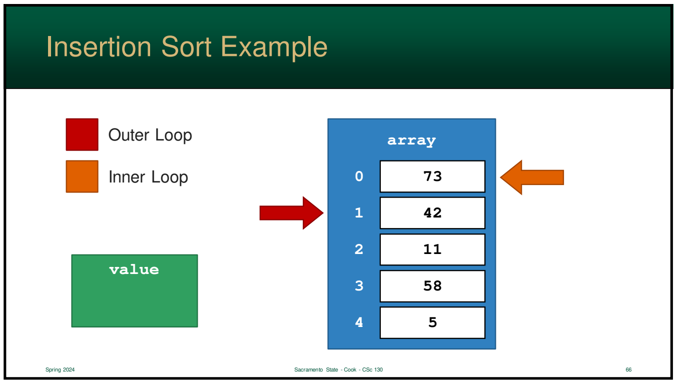


Insertion Sort Example

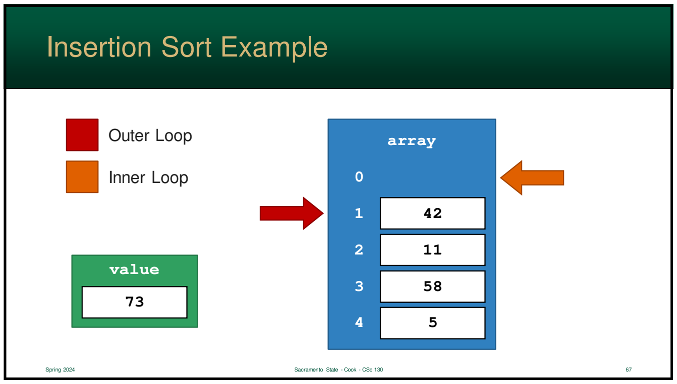
🟥 Outer Loop

🟧 Inner Loop

value

array	
0	73
1	42
2	11
3	58
4	5

66



Insertion Sort Example

🟥 Outer Loop

🟧 Inner Loop

value
73

	array
0	
1	42
2	11
3	58
4	5

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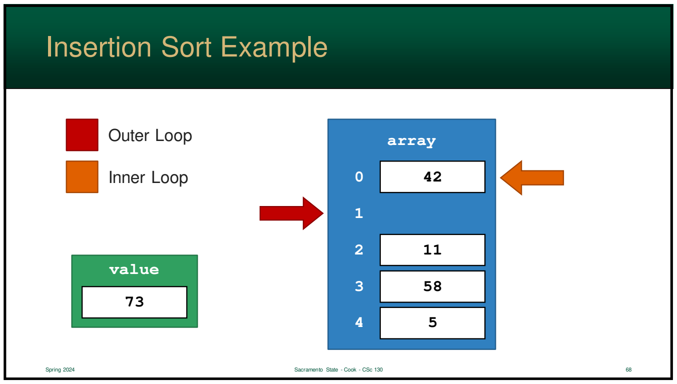
67



Insertion Sort Example

🟥 Outer Loop

🟧 Inner Loop

value
73

	array
0	42
1	
2	11
3	58
4	5

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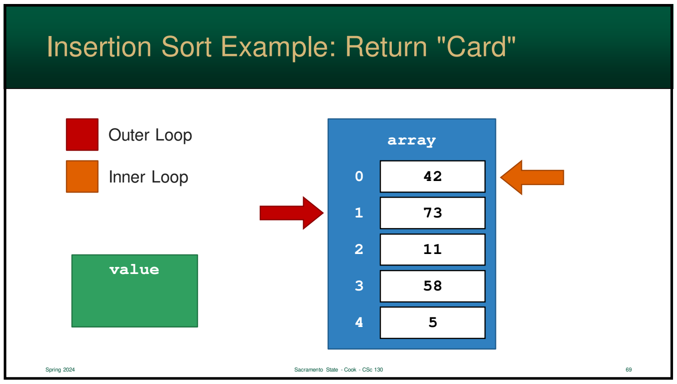
68



Insertion Sort Example: Return "Card"

🟥 Outer Loop

🟧 Inner Loop

value

	array
0	42
1	73
2	11
3	58
4	5

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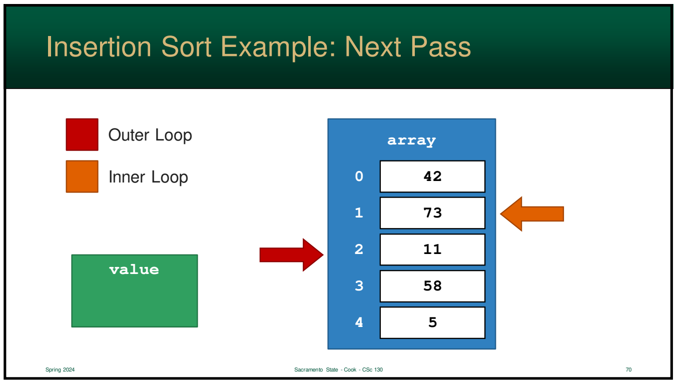
69



Insertion Sort Example: Next Pass

🟥 Outer Loop

🟧 Inner Loop

value

	array
0	42
1	73
2	11
3	58
4	5

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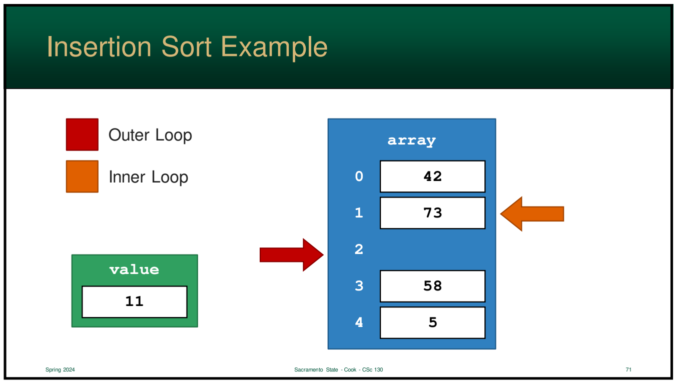
70



Insertion Sort Example

🟥 Outer Loop

🟧 Inner Loop

value
11

	array
0	42
1	73
2	
3	58
4	5

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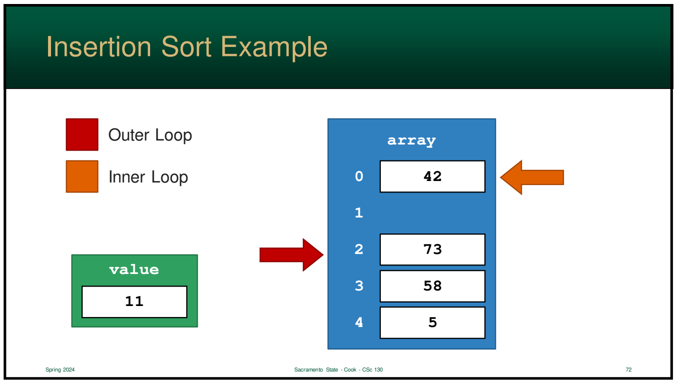
71



Insertion Sort Example

🟥 Outer Loop

🟧 Inner Loop

value
11

	array
0	42
1	
2	73
3	58
4	5

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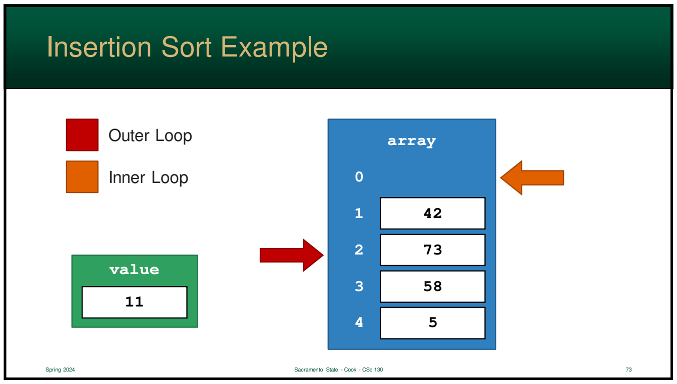
72



Insertion Sort Example

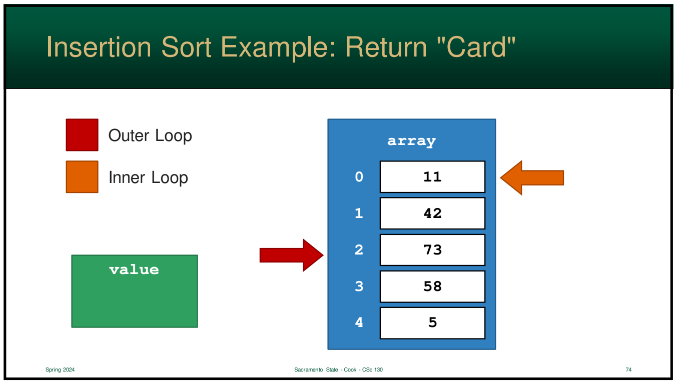
🟥 Outer Loop

🟧 Inner Loop

array	
0	
1	42
2	73
3	58
4	5

value: 11

73



Insertion Sort Example: Return "Card"

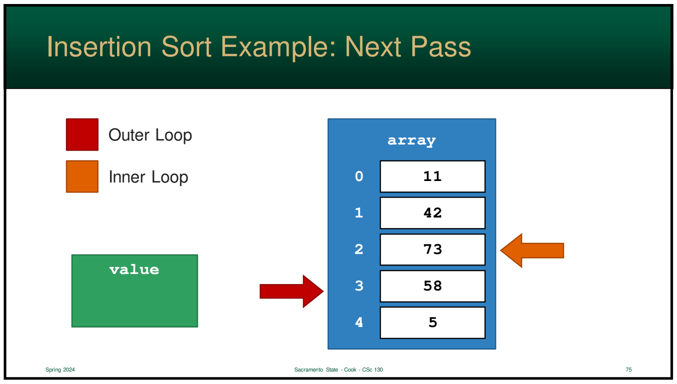
🟥 Outer Loop

🟧 Inner Loop

array	
0	11
1	42
2	73
3	58
4	5

value

74



Insertion Sort Example: Next Pass

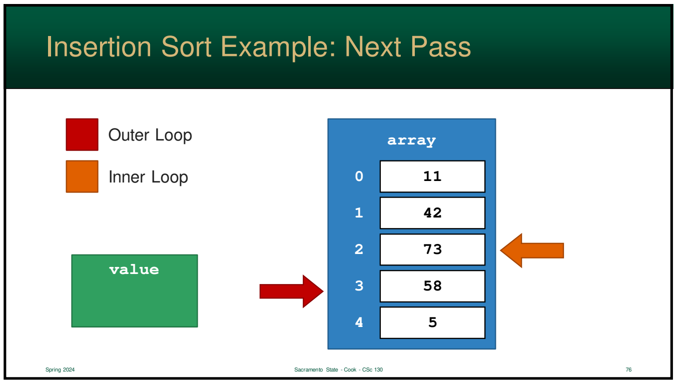
🟥 Outer Loop

🟧 Inner Loop

array	
0	11
1	42
2	73
3	58
4	5

value

75



Insertion Sort Example: Next Pass

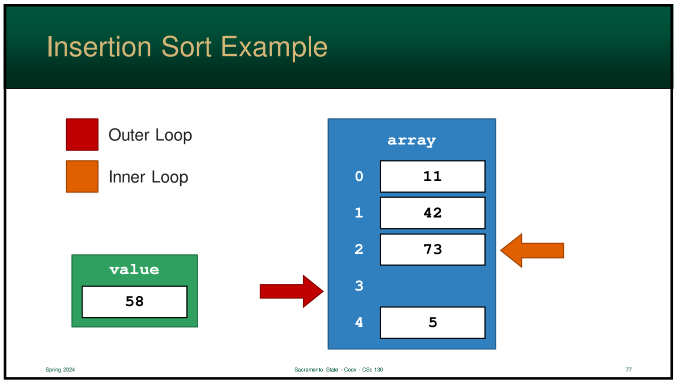
🟥 Outer Loop

🟧 Inner Loop

array	
0	11
1	42
2	73
3	58
4	5

value

76



Insertion Sort Example

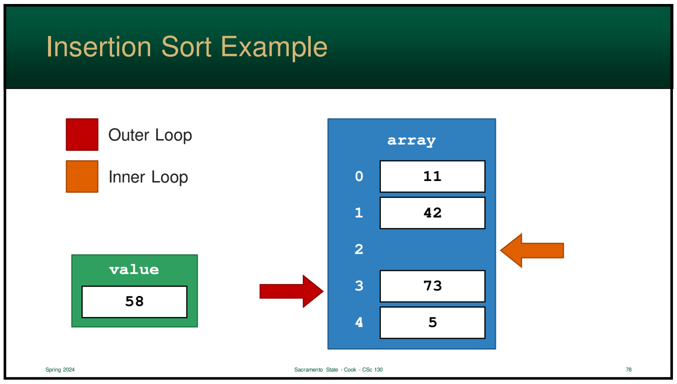
🟥 Outer Loop

🟧 Inner Loop

array	
0	11
1	42
2	73
3	
4	5

value: 58

77



Insertion Sort Example

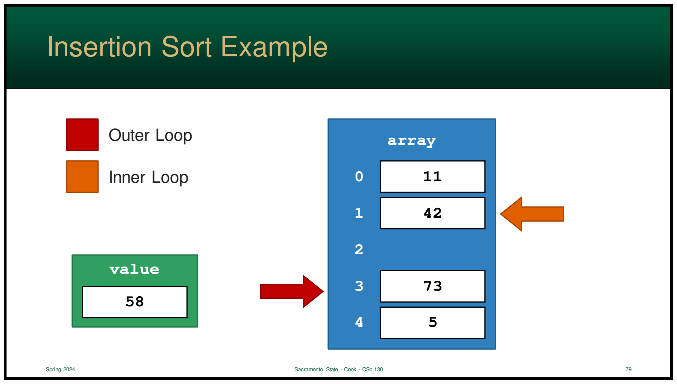
🟥 Outer Loop

🟧 Inner Loop

array	
0	11
1	42
2	
3	73
4	5

value: 58

78



Insertion Sort Example

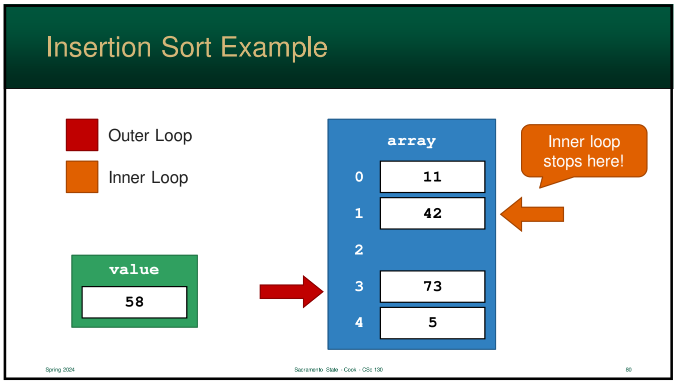
🟥 Outer Loop

🟧 Inner Loop

value
58

array	
0	11
1	42
2	
3	73
4	5

79



Insertion Sort Example

🟥 Outer Loop

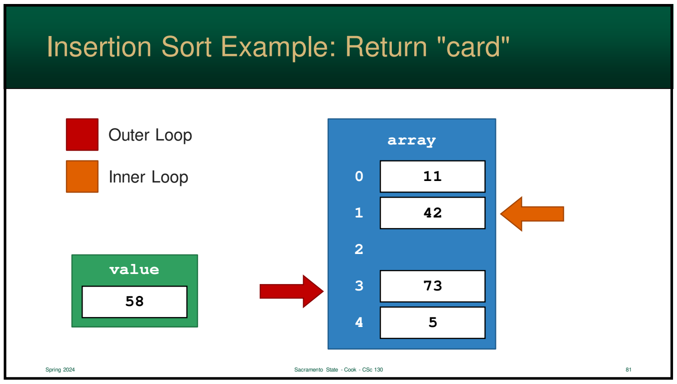
🟧 Inner Loop

value
58

array	
0	11
1	42
2	
3	73
4	5

Inner loop stops here!

80



Insertion Sort Example: Return "card"

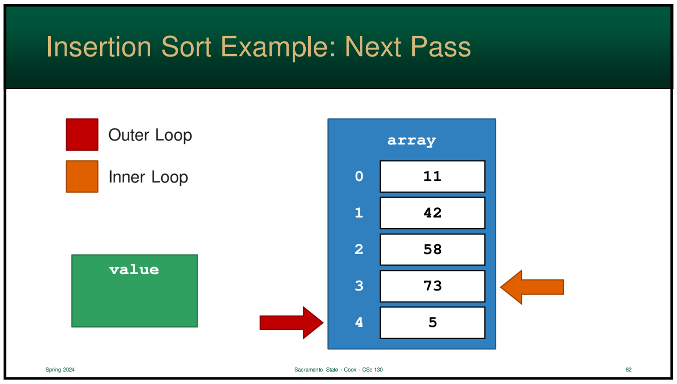
🟥 Outer Loop

🟧 Inner Loop

value
58

array	
0	11
1	42
2	
3	73
4	5

81



Insertion Sort Example: Next Pass

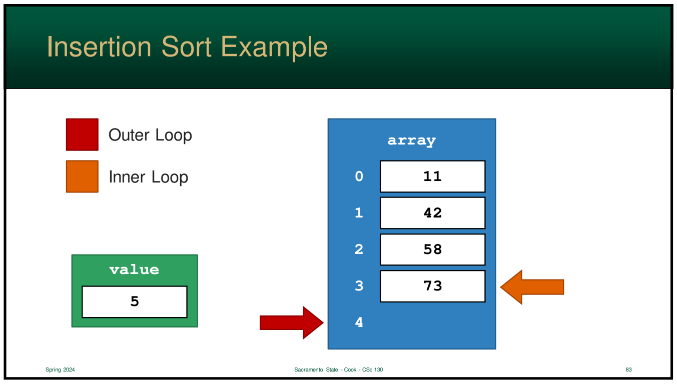
🟥 Outer Loop

🟧 Inner Loop

value

array	
0	11
1	42
2	58
3	73
4	5

82



Insertion Sort Example

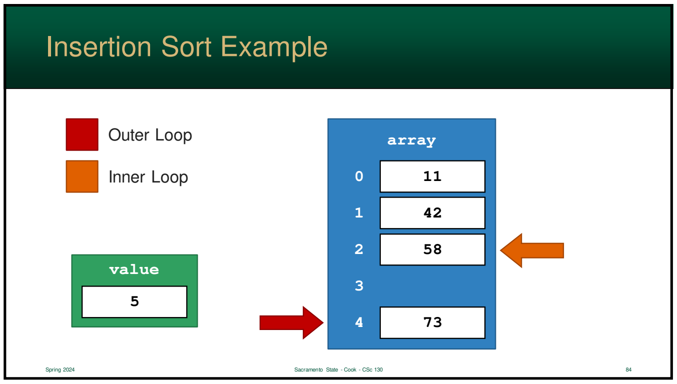
🟥 Outer Loop

🟧 Inner Loop

value
5

array	
0	11
1	42
2	58
3	73
4	

83



Insertion Sort Example

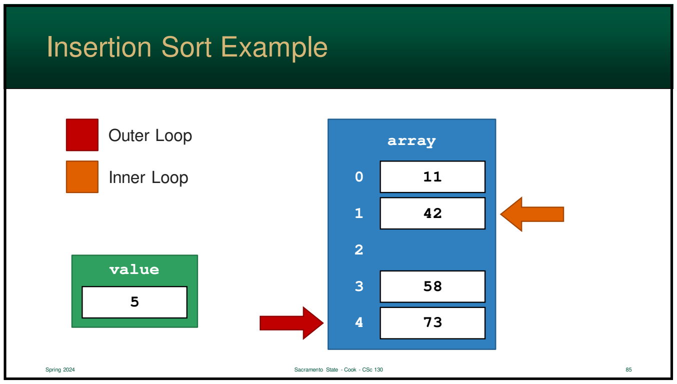
🟥 Outer Loop

🟧 Inner Loop

value
5

array	
0	11
1	42
2	58
3	
4	73

84



Insertion Sort Example

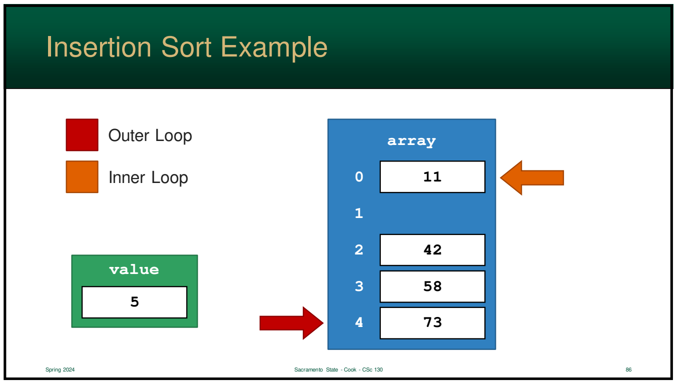
🟥 Outer Loop

🟧 Inner Loop

array	
0	11
1	42
2	
3	58
4	73

value: 5

85



Insertion Sort Example

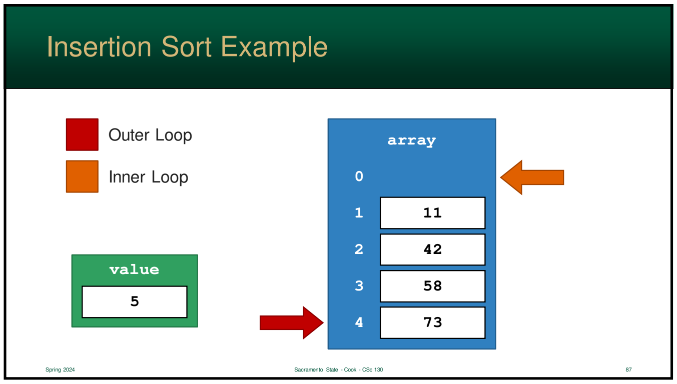
🟥 Outer Loop

🟧 Inner Loop

array	
0	11
1	
2	42
3	58
4	73

value: 5

86



Insertion Sort Example

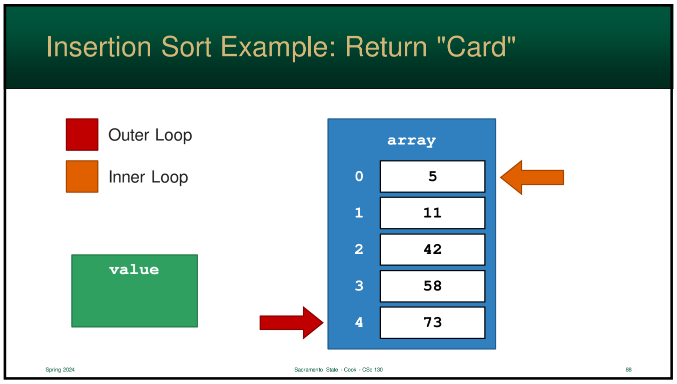
🟥 Outer Loop

🟧 Inner Loop

array	
0	
1	11
2	42
3	58
4	73

value: 5

87



Insertion Sort Example: Return "Card"

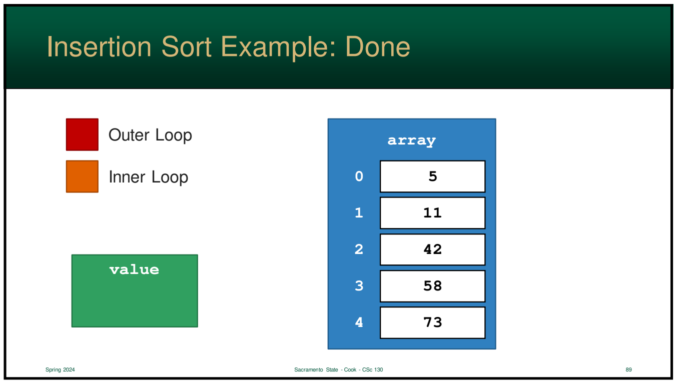
🟥 Outer Loop

🟧 Inner Loop

array	
0	5
1	11
2	42
3	58
4	73

value

88



Insertion Sort Example: Done

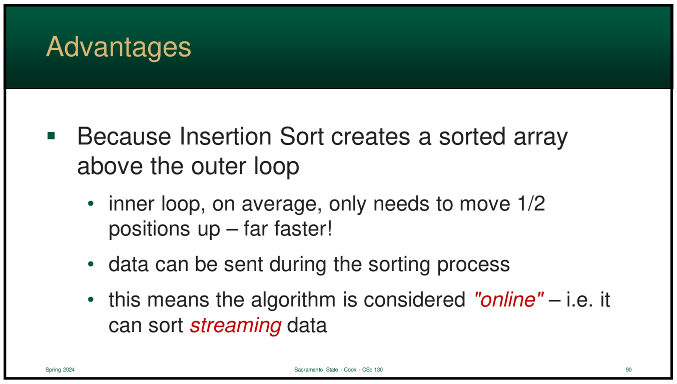
🟥 Outer Loop

🟧 Inner Loop

array	
0	5
1	11
2	42
3	58
4	73

value

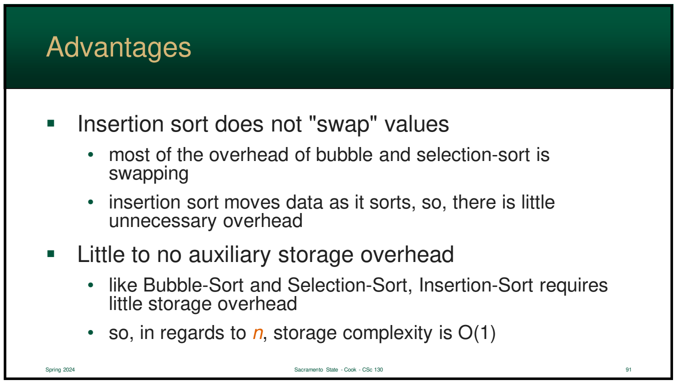
89



Advantages

- Because Insertion Sort creates a sorted array above the outer loop
  - inner loop, on average, only needs to move 1/2 positions up – far faster!
  - data can be sent during the sorting process
  - this means the algorithm is considered *"online"* – i.e. it can sort *streaming* data

90



Advantages

- Insertion sort does not "swap" values
  - most of the overhead of bubble and selection-sort is swapping
  - insertion sort moves data as it sorts, so, there is little unnecessary overhead
- Little to no auxiliary storage overhead
  - like Bubble-Sort and Selection-Sort, Insertion-Sort requires little storage overhead
  - so, in regards to *n*, storage complexity is O(1)

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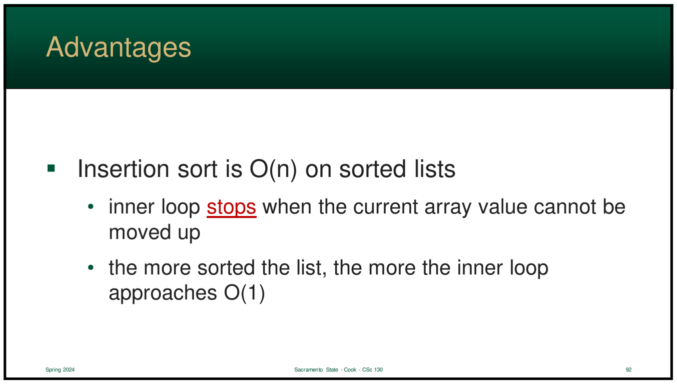
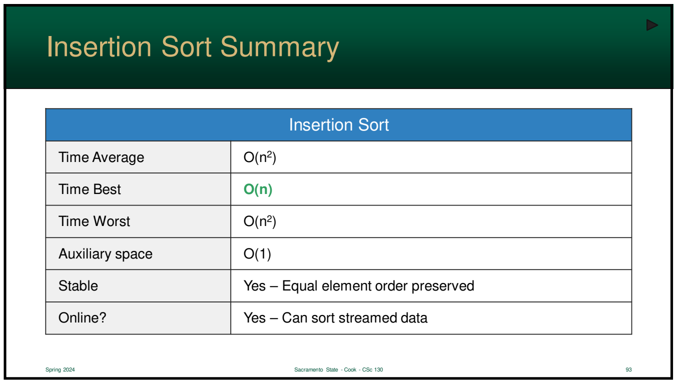


Advantages

- Insertion sort is O(n) on sorted lists
  - inner loop stops when the current array value cannot be moved up
  - the more sorted the list, the more the inner loop approaches O(1)

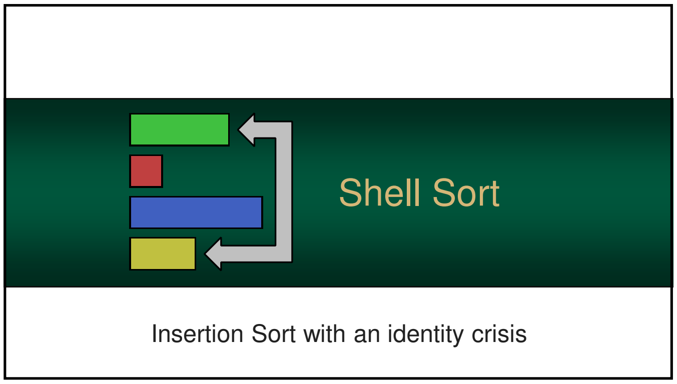
92



Insertion Sort Summary

Insertion Sort	
Time Average	O(n^2)
Time Best	**O(n)**
Time Worst	O(n^2)
Auxiliary space	O(1)
Stable	Yes – Equal element order preserved
Online?	Yes – Can sort streamed data

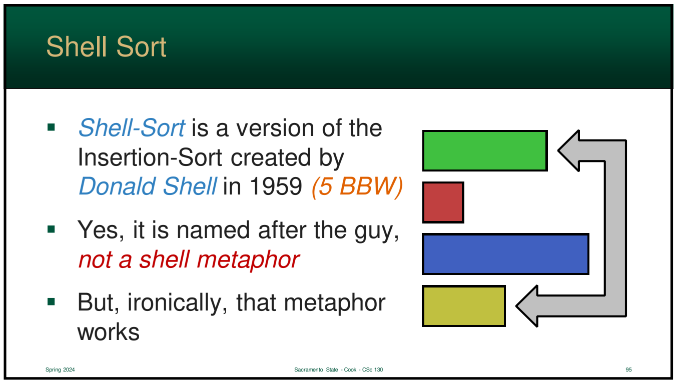
93



Shell Sort

Insertion Sort with an identity crisis

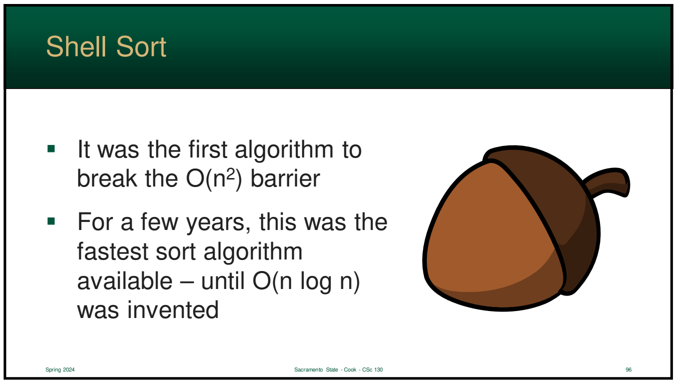
94



Shell Sort

- *Shell-Sort* is a version of the Insertion-Sort created by *Donald Shell* in 1959 *(5 BBW)*
- Yes, it is named after the guy, *not a shell metaphor*
- But, ironically, that metaphor works

95



Shell Sort

- It was the first algorithm to break the O(n^2) barrier
- For a few years, this was the fastest sort algorithm available – until O(n log n) was invented

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What is Going On?

- With insertion sort, each time we insert an element, the rest are moved one step closer to where they belong
- Can we move elements a larger distance than just one?
- **Yes…** Shell Sort works like Insertion Sort, but works on elements at large distances
- This distance is called the *gap*

What's Going On?

- Gap changes with each outer loop iteration
  - the distance between comparisons decreases as the sorting algorithm runs
  - in the last iteration, the gap is 1
  - so, at that point, adjacent elements are compared – so it is a regular Insertion Sort
- Shell Sort is also known as a "diminishing increment sort"

Sorting "Shells"

- Shell Sort orders elements that are spaced a relative distance from each other
- So, the red cells above are sorted relative to each other, as are the yellow, green, and blue elements

46	50	92	83	11	36	47	76	53	49	22	3

Sorting "Shells"

- The decreasing gaps are a sequence
- The notation $h_1, h_2, h_3, . . . , h_t$ represents a sequence of increasing integer values which will be used (from right to left)
- Any sequence works if it $h_n > h_{n-1}$ and $h_1 = 1$

46	50	92	83	11	36	47	76	53	49	22	3

Each Shell is Sorted

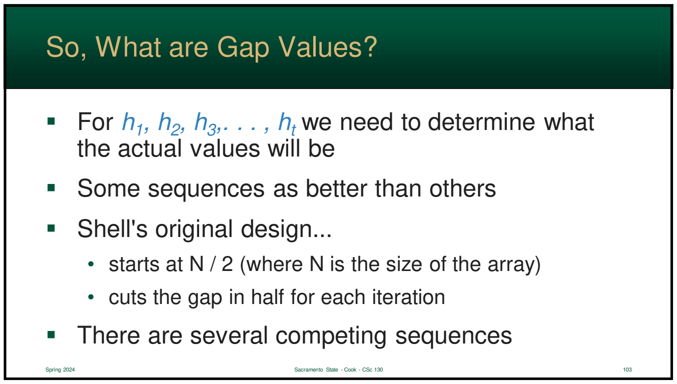
- *h_k-sorted array* - all elements with gap h_k are sorted relative to each other
- For each i, we have array[i] ≤ array[i + h_k]
- All elements spaced h_k apart will be sorted

46	50	92	83	11	36	47	76	53	49	22	3

Each Shell is Sorted

- Shell-Sort only works because an array that is h_k-sorted…
- …remains h_k-sorted when h_{k-1}-sorted.

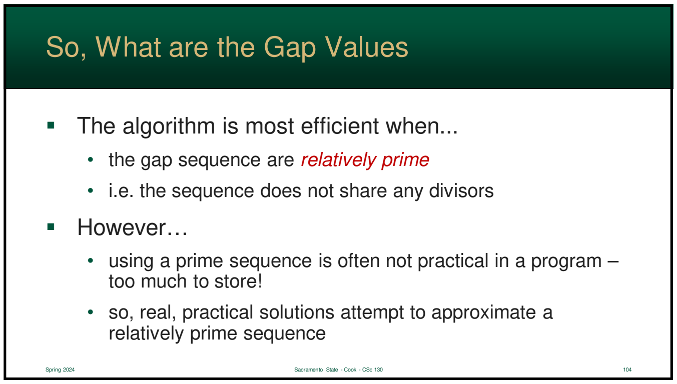
11	36	22	3	46	49	47	76	53	50	92	83



So, What are Gap Values?

- For $h_1, h_2, h_3, . . . , h_t$ we need to determine what the actual values will be
- Some sequences as better than others
- Shell's original design...
  - starts at N / 2 (where N is the size of the array)
  - cuts the gap in half for each iteration
- There are several competing sequences

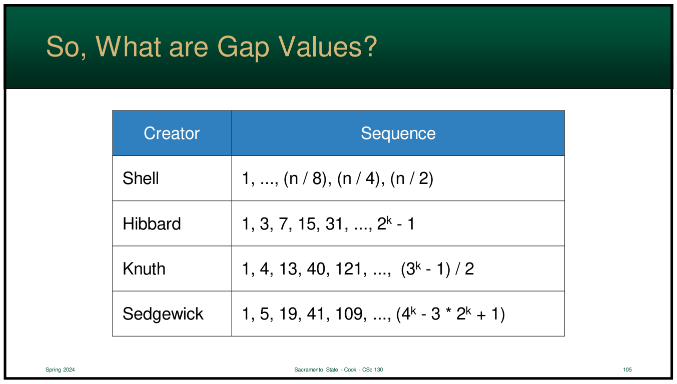
103



So, What are the Gap Values

- The algorithm is most efficient when...
  - the gap sequence are *relatively prime*
  - i.e. the sequence does not share any divisors
- However…
  - using a prime sequence is often not practical in a program – too much to store!
  - so, real, practical solutions attempt to approximate a relatively prime sequence

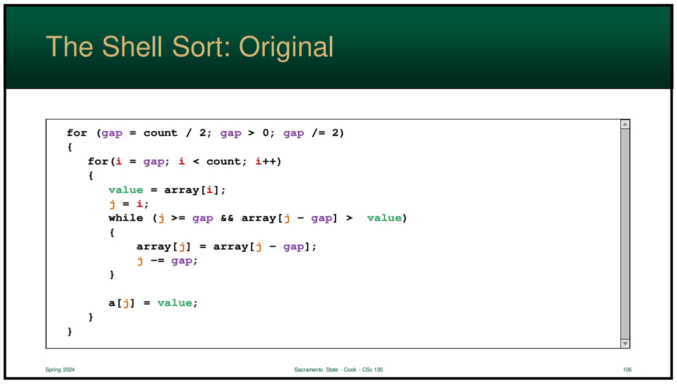
104



So, What are Gap Values?

Creator	Sequence
Shell	1, ..., (n / 8), (n / 4), (n / 2)
Hibbard	1, 3, 7, 15, 31, ..., 2^k - 1
Knuth	1, 4, 13, 40, 121, ..., (3^k - 1) / 2
Sedgewick	1, 5, 19, 41, 109, ..., (4^k - 3 * 2^k + 1)

105

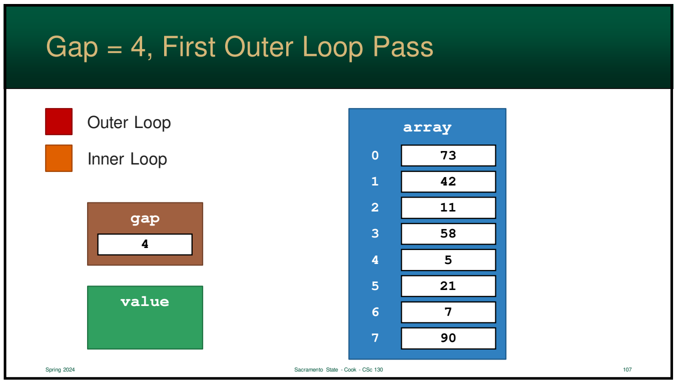


The Shell Sort: Original

```
for (gap = count / 2; gap > 0; gap /= 2)
{
   for(i = gap; i < count; i++)
   {
      value = array[i];
      j = i;
      while (j >= gap && array[j - gap] >  value)
      {
          array[j] = array[j - gap];
          j -= gap;
      }

      a[j] = value;
   }
}
```

106



Gap = 4, First Outer Loop Pass

🟥 Outer Loop

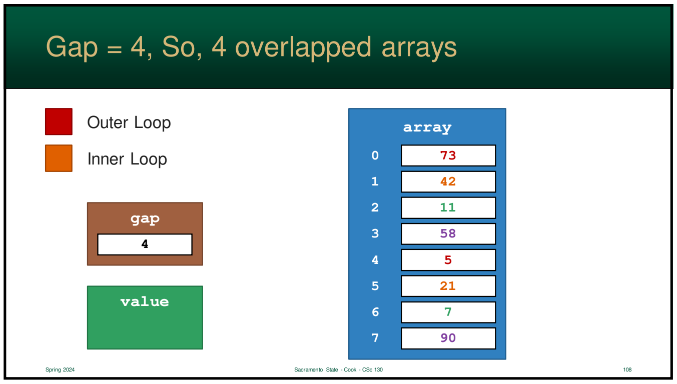
🟧 Inner Loop

gap: 4

value

array	
0	73
1	42
2	11
3	58
4	5
5	21
6	7
7	90

107



Gap = 4, So, 4 overlapped arrays

🟥 Outer Loop

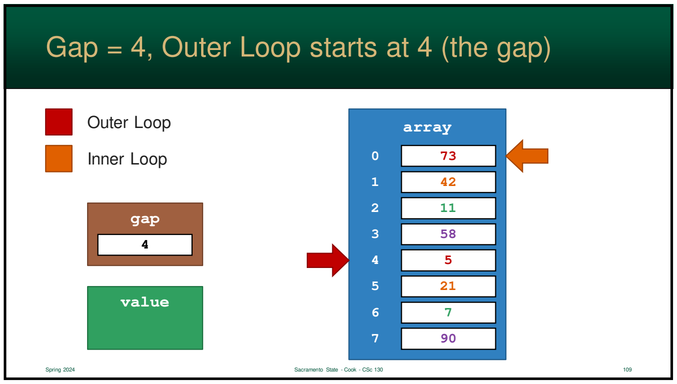
🟧 Inner Loop

gap: 4

value

array	
0	73
1	42
2	11
3	58
4	5
5	21
6	7
7	90

108



Gap = 4, Outer Loop starts at 4 (the gap)

🟥 Outer Loop

🟧 Inner Loop

gap
4

value

		array	
	0	73	←
	1	42	
	2	11	
	3	58	
→	4	5	
	5	21	
	6	7	
	7	90	

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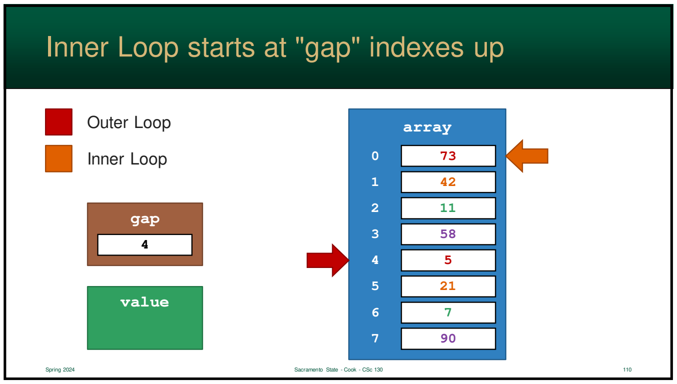
109



Inner Loop starts at "gap" indexes up

🟥 Outer Loop

🟧 Inner Loop

gap
4

value

		array	
	0	73	←
	1	42	
	2	11	
	3	58	
→	4	5	
	5	21	
	6	7	
	7	90	

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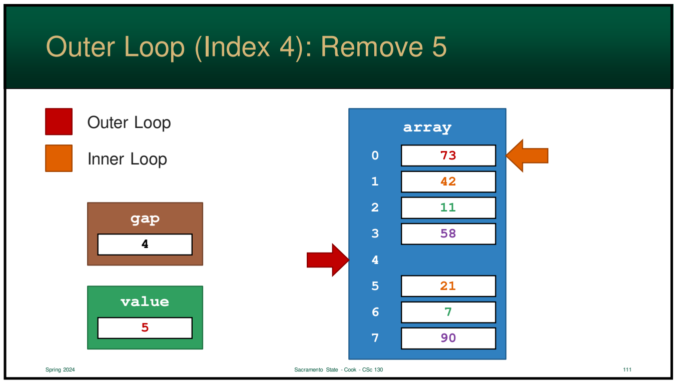
110



Outer Loop (Index 4): Remove 5

🟥 Outer Loop

🟧 Inner Loop

gap
4

value
5

		array	
	0	73	←
	1	42	
	2	11	
	3	58	
→	4		
	5	21	
	6	7	
	7	90	

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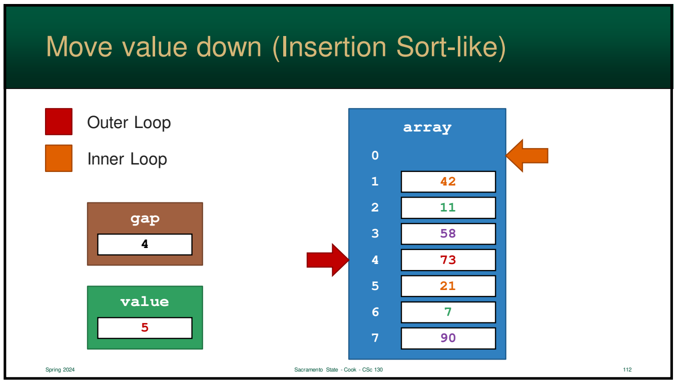
111



Move value down (Insertion Sort-like)

🟥 Outer Loop

🟧 Inner Loop

gap
4

value
5

		array	
	0		←
	1	42	
	2	11	
	3	58	
→	4	73	
	5	21	
	6	7	
	7	90	

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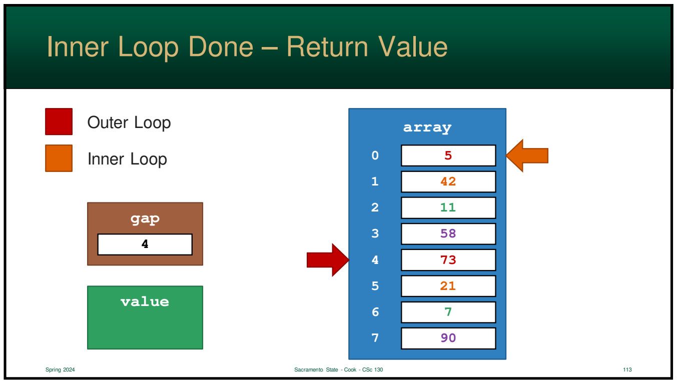
112



Inner Loop Done – Return Value

🟥 Outer Loop

🟧 Inner Loop

gap
4

value

		array	
	0	5	←
	1	42	
	2	11	
	3	58	
→	4	73	
	5	21	
	6	7	
	7	90	

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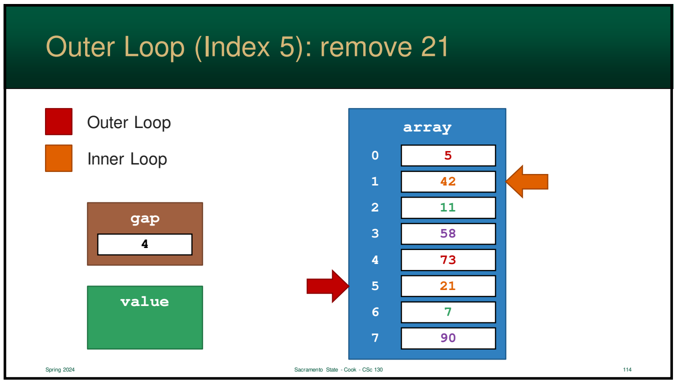
113



Outer Loop (Index 5): remove 21

🟥 Outer Loop

🟧 Inner Loop

gap
4

value

		array	
	0	5	
	1	42	←
	2	11	
	3	58	
	4	73	
→	5	21	
	6	7	
	7	90	

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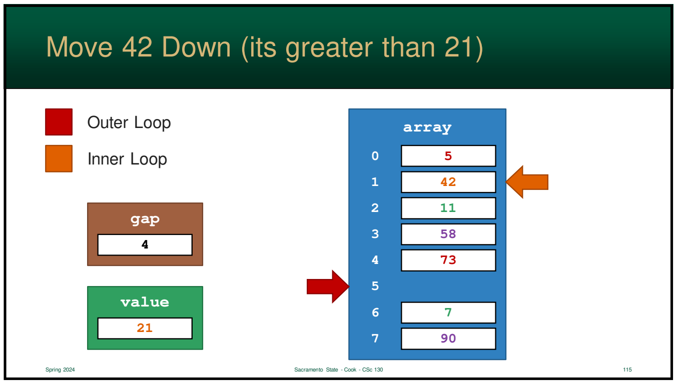
114



Move 42 Down (its greater than 21)

🟥 Outer Loop

🟧 Inner Loop

gap: 4

value: 21

array	
0	5
1	42
2	11
3	58
4	73
5	
6	7
7	90

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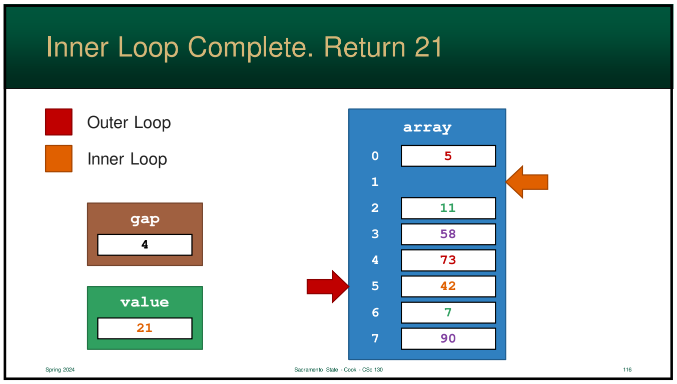
115



Inner Loop Complete. Return 21

🟥 Outer Loop

🟧 Inner Loop

gap: 4

value: 21

array	
0	5
1	
2	11
3	58
4	73
5	42
6	7
7	90

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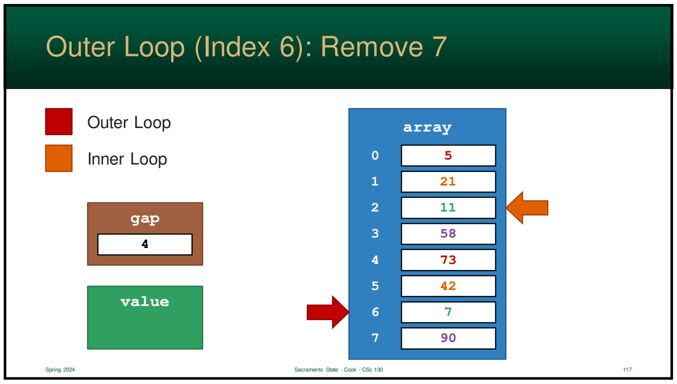
116



Outer Loop (Index 6): Remove 7

🟥 Outer Loop

🟧 Inner Loop

gap: 4

value:

array	
0	5
1	21
2	11
3	58
4	73
5	42
6	7
7	90

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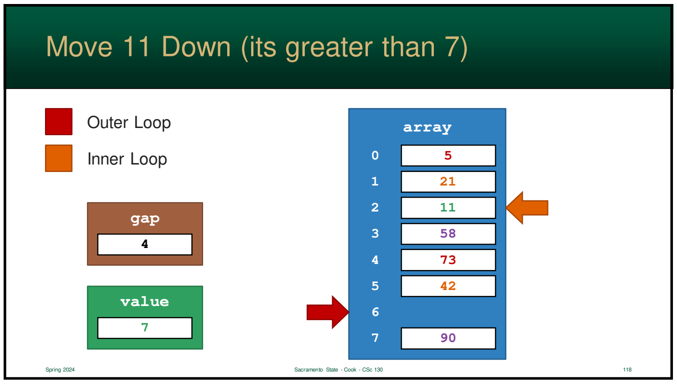
117



Move 11 Down (its greater than 7)

🟥 Outer Loop

🟧 Inner Loop

gap: 4

value: 7

array	
0	5
1	21
2	11
3	58
4	73
5	42
6	
7	90

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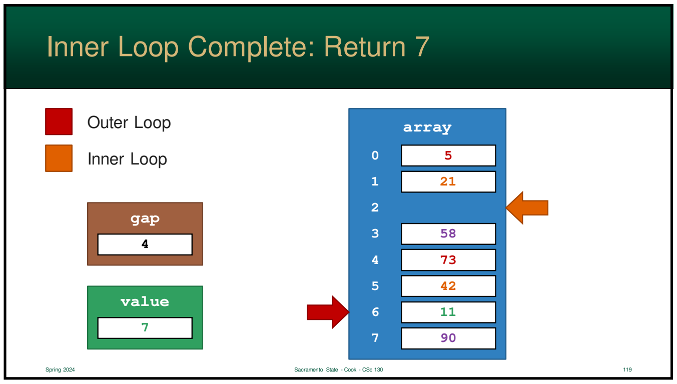
118



Inner Loop Complete: Return 7

🟥 Outer Loop

🟧 Inner Loop

gap: 4

value: 7

array	
0	5
1	21
2	
3	58
4	73
5	42
6	11
7	90

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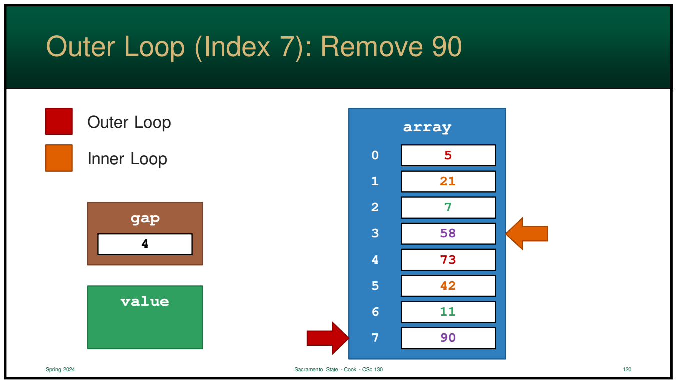
119



Outer Loop (Index 7): Remove 90

🟥 Outer Loop

🟧 Inner Loop

gap: 4

value:

array	
0	5
1	21
2	7
3	58
4	73
5	42
6	11
7	90

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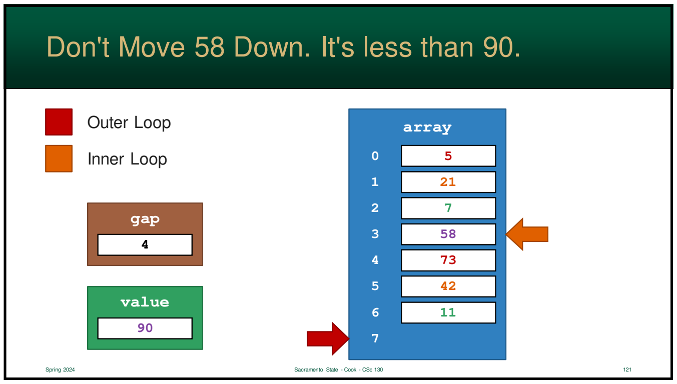
120



Don't Move 58 Down. It's less than 90.

🟥 Outer Loop

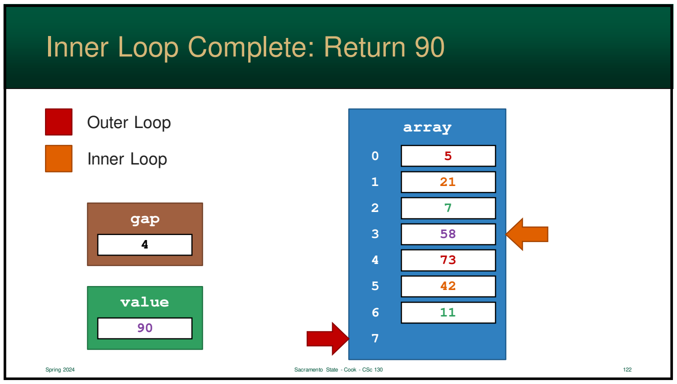
🟧 Inner Loop

gap: 4

value: 90

array	
0	5
1	21
2	7
3	58
4	73
5	42
6	11
7	

121



Inner Loop Complete: Return 90

🟥 Outer Loop

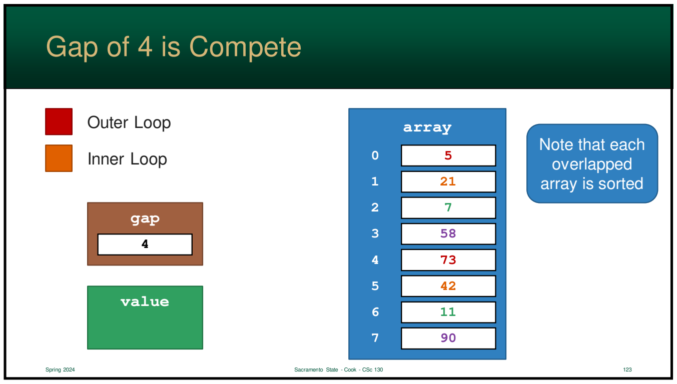
🟧 Inner Loop

gap: 4

value: 90

array	
0	5
1	21
2	7
3	58
4	73
5	42
6	11
7	

122



Gap of 4 is Compete

🟥 Outer Loop

🟧 Inner Loop

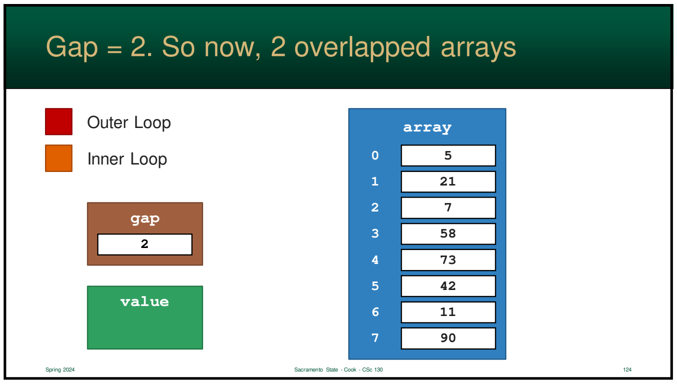
gap: 4

value:

array	
0	5
1	21
2	7
3	58
4	73
5	42
6	11
7	90

Note that each overlapped array is sorted

123



Gap = 2. So now, 2 overlapped arrays

🟥 Outer Loop

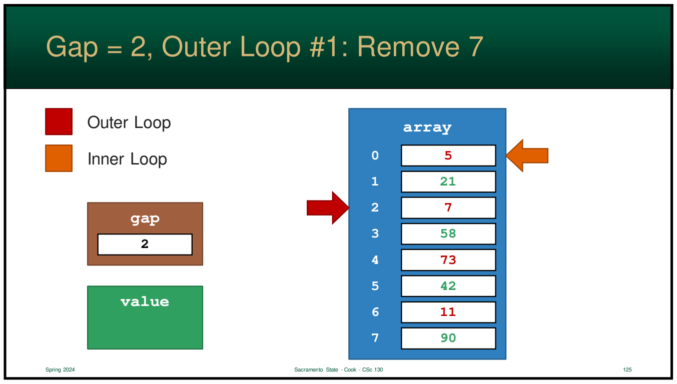
🟧 Inner Loop

gap: 2

value:

array	
0	5
1	21
2	7
3	58
4	73
5	42
6	11
7	90

124



Gap = 2, Outer Loop #1: Remove 7

🟥 Outer Loop

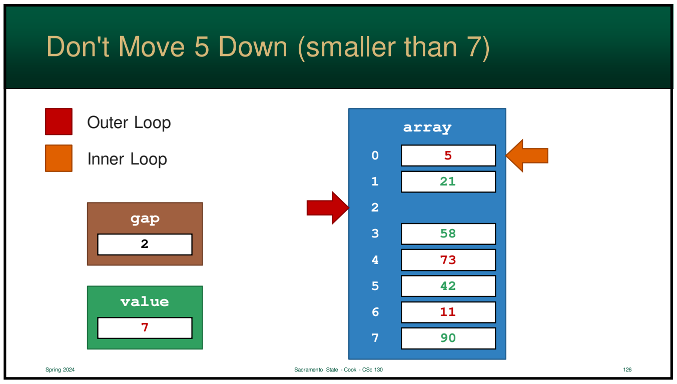
🟧 Inner Loop

gap: 2

value:

array	
0	5
1	21
2	7
3	58
4	73
5	42
6	11
7	90

125



Don't Move 5 Down (smaller than 7)

🟥 Outer Loop

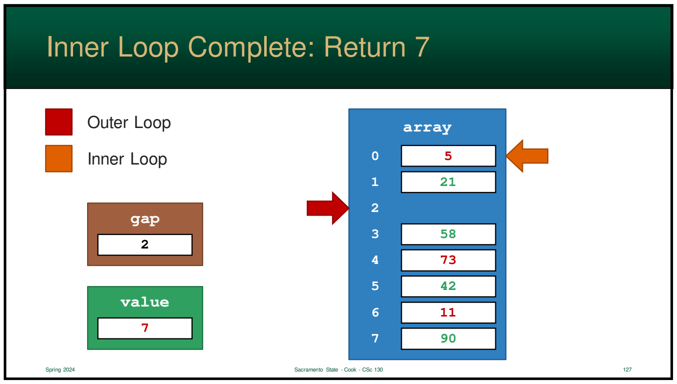
🟧 Inner Loop

gap: 2

value: 7

array	
0	5
1	21
2	
3	58
4	73
5	42
6	11
7	90

126



Inner Loop Complete: Return 7

🟥 Outer Loop

🟧 Inner Loop

gap

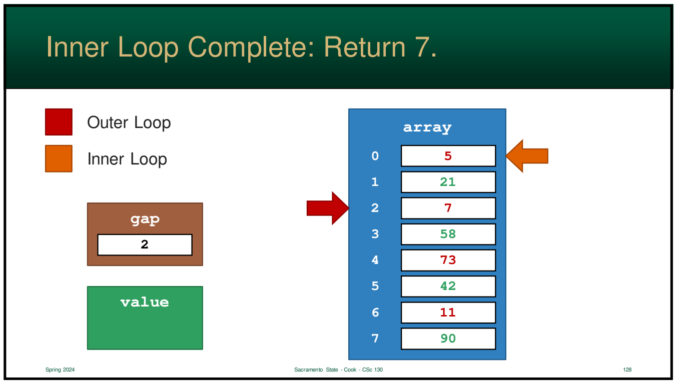
2

value

7

	array
0	5
1	21
2	
3	58
4	73
5	42
6	11
7	90

127



Inner Loop Complete: Return 7.

🟥 Outer Loop

🟧 Inner Loop

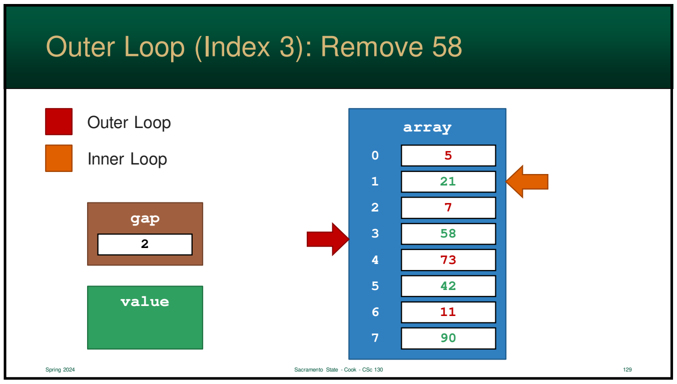
gap

2

value

	array
0	5
1	21
2	7
3	58
4	73
5	42
6	11
7	90

128



Outer Loop (Index 3): Remove 58

🟥 Outer Loop

🟧 Inner Loop

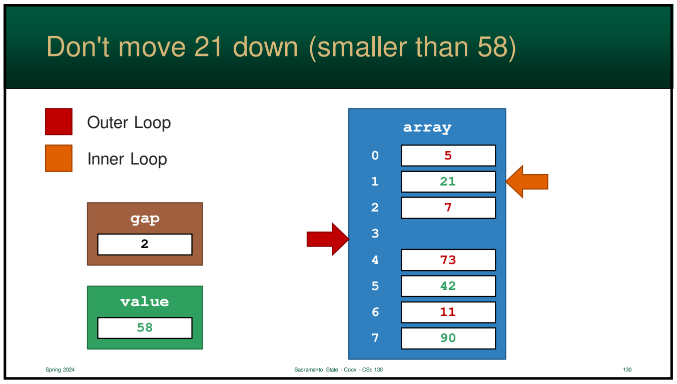
gap

2

value

	array
0	5
1	21
2	7
3	58
4	73
5	42
6	11
7	90

129



Don't move 21 down (smaller than 58)

🟥 Outer Loop

🟧 Inner Loop

gap

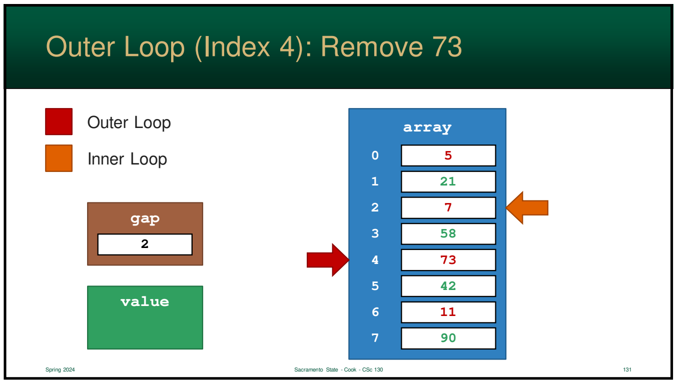
2

value

58

	array
0	5
1	21
2	7
3	
4	73
5	42
6	11
7	90

130



Outer Loop (Index 4): Remove 73

🟥 Outer Loop

🟧 Inner Loop

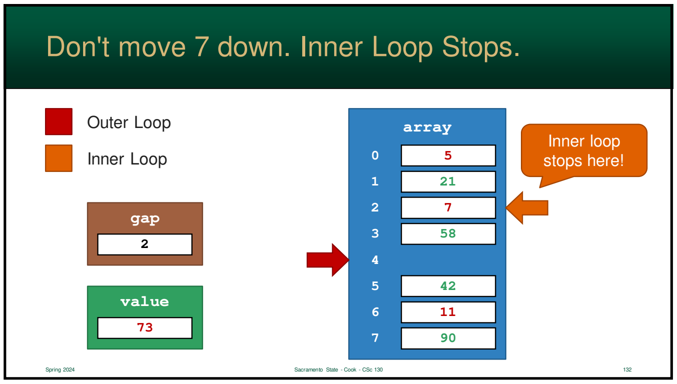
gap

2

value

	array
0	5
1	21
2	7
3	58
4	73
5	42
6	11
7	90

131



Don't move 7 down. Inner Loop Stops.

🟥 Outer Loop

🟧 Inner Loop

gap

2

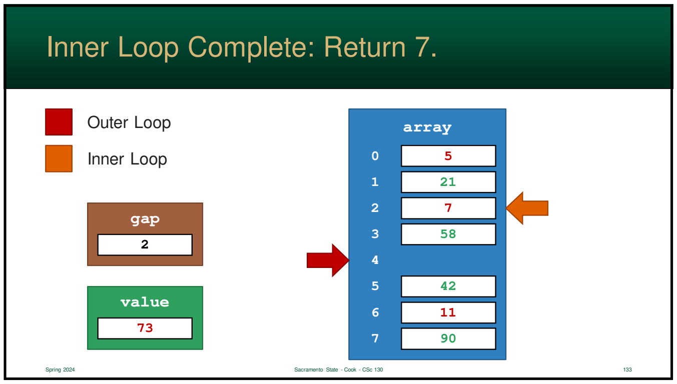
value

73

	array
0	5
1	21
2	7
3	58
4	
5	42
6	11
7	90

Inner loop stops here!

132



Inner Loop Complete: Return 7.

🟥 Outer Loop

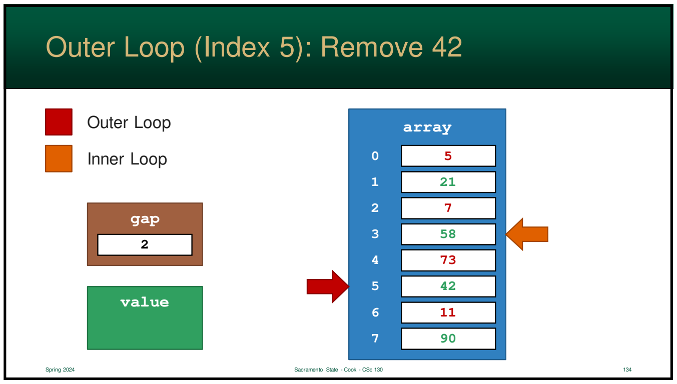
🟧 Inner Loop

gap: 2

value: 73

	array
0	5
1	21
2	7
3	58
4	
5	42
6	11
7	90

133



Outer Loop (Index 5): Remove 42

🟥 Outer Loop

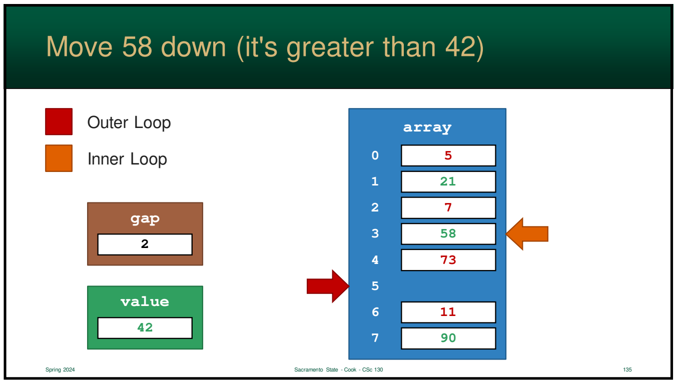
🟧 Inner Loop

gap: 2

value:

	array
0	5
1	21
2	7
3	58
4	73
5	42
6	11
7	90

134



Move 58 down (it's greater than 42)

🟥 Outer Loop

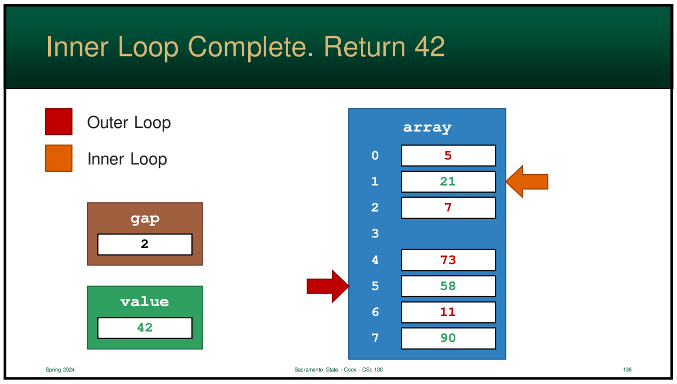
🟧 Inner Loop

gap: 2

value: 42

	array
0	5
1	21
2	7
3	58
4	73
5	
6	11
7	90

135



Inner Loop Complete. Return 42

🟥 Outer Loop

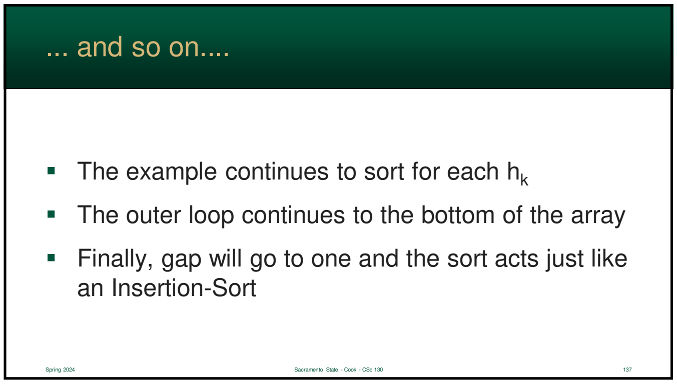
🟧 Inner Loop

gap: 2

value: 42

	array
0	5
1	21
2	7
3	
4	73
5	58
6	11
7	90

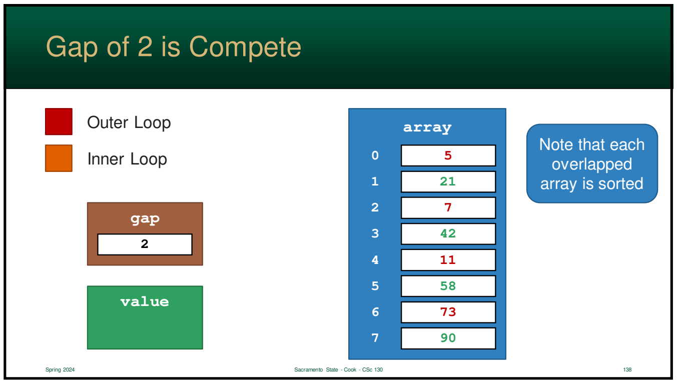
136



... and so on....

- The example continues to sort for each h_k
- The outer loop continues to the bottom of the array
- Finally, gap will go to one and the sort acts just like an Insertion-Sort

137



Gap of 2 is Compete

🟥 Outer Loop

🟧 Inner Loop

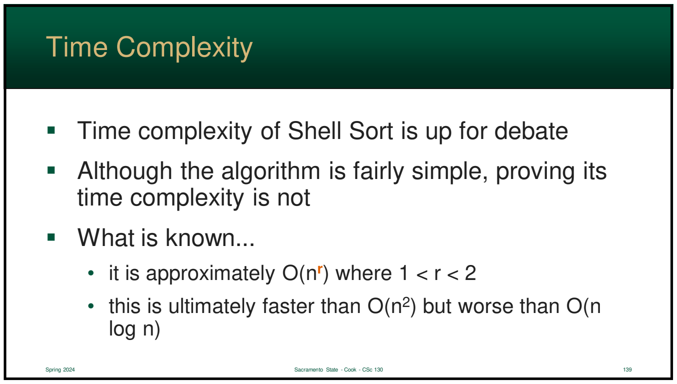
gap: 2

value:

	array
0	5
1	21
2	7
3	42
4	11
5	58
6	73
7	90

Note that each overlapped array is sorted

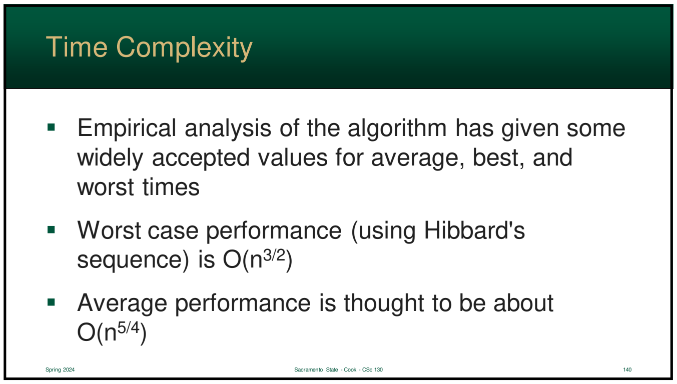
138



Time Complexity

- Time complexity of Shell Sort is up for debate
- Although the algorithm is fairly simple, proving its time complexity is not
- What is known...
  - it is approximately $O(n^r)$ where $1 < r < 2$
  - this is ultimately faster than $O(n^2)$ but worse than O(n log n)

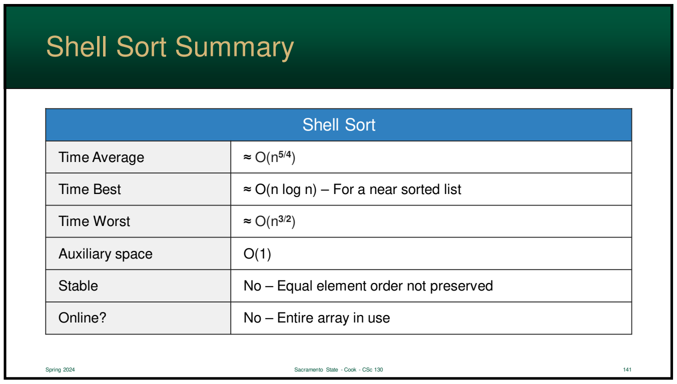
139



Time Complexity

- Empirical analysis of the algorithm has given some widely accepted values for average, best, and worst times
- Worst case performance (using Hibbard's sequence) is $O(n^{3/2})$
- Average performance is thought to be about $O(n^{5/4})$

140



Shell Sort Summary

Shell Sort	
Time Average	≈ $O(n^{5/4})$
Time Best	≈ O(n log n) – For a near sorted list
Time Worst	≈ $O(n^{3/2})$
Auxiliary space	O(1)
Stable	No – Equal element order not preserved
Online?	No – Entire array in use

141