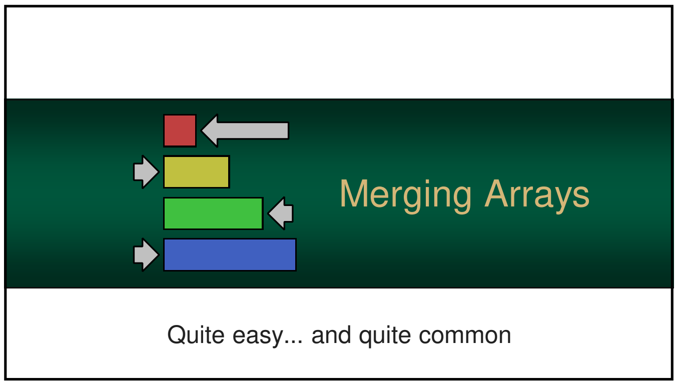
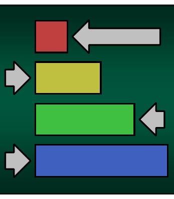
Recursive Sorting

Part 6

1





Merging Arrays

Quite easy... and quite common

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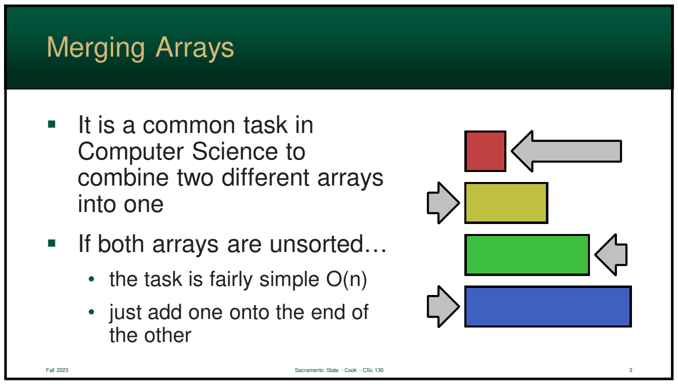
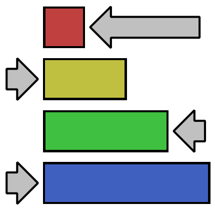


Merging Arrays

- It is a common task in Computer Science to combine two different arrays into one
- If both arrays are unsorted…
  - the task is fairly simple O(n)
  - just add one onto the end of the other



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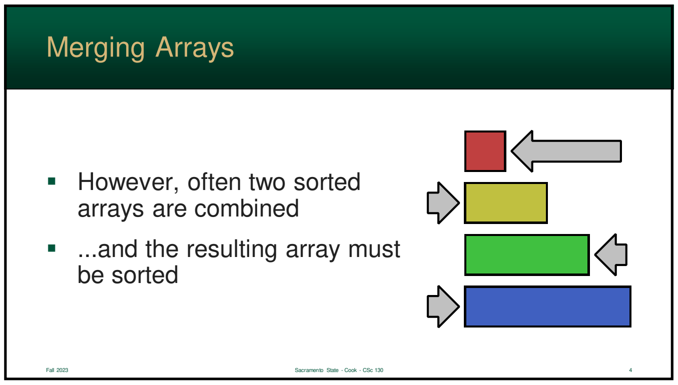
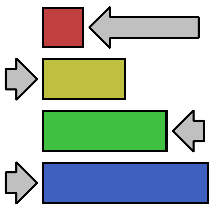


Merging Arrays

- However, often two sorted arrays are combined
- ...and the resulting array must be sorted



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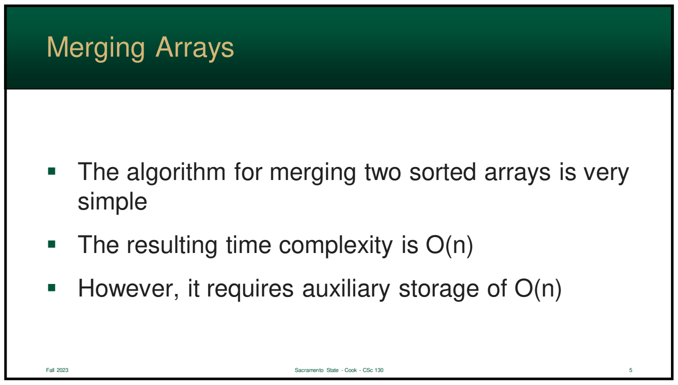

Merging Arrays

- The algorithm for merging two sorted arrays is very simple
- The resulting time complexity is O(n)
- However, it requires auxiliary storage of O(n)

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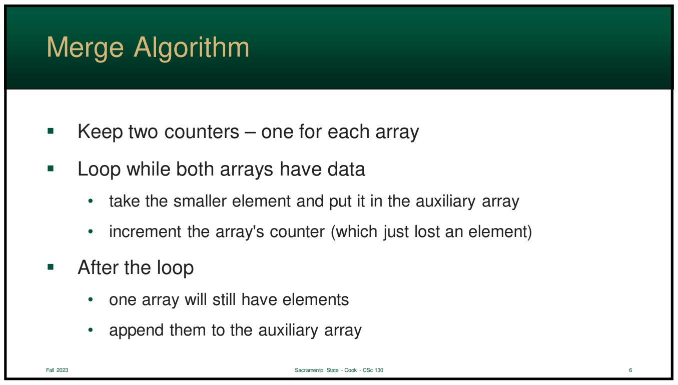

Merge Algorithm

- Keep two counters – one for each array
- Loop while both arrays have data
  - take the smaller element and put it in the auxiliary array
  - increment the array's counter (which just lost an element)
- After the loop
  - one array will still have elements
  - append them to the auxiliary array

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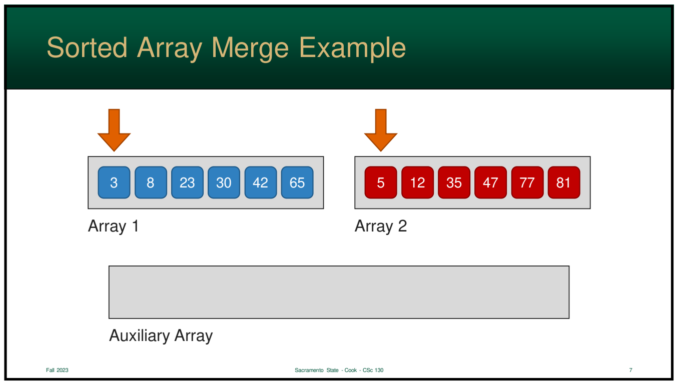

Sorted Array Merge Example

3 8 23 30 42 65

Array 1

5 12 35 47 77 81

Array 2

Auxiliary Array

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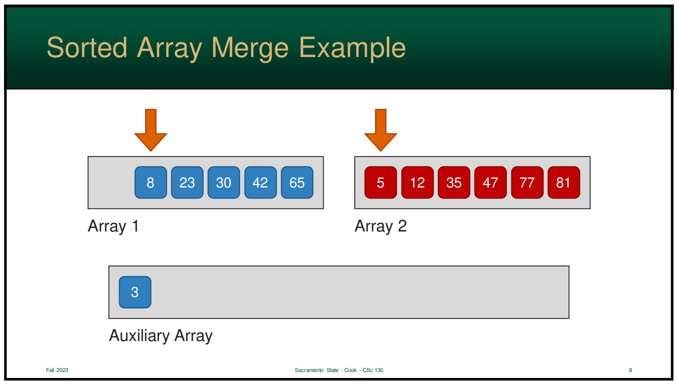

Sorted Array Merge Example

8 23 30 42 65

Array 1

5 12 35 47 77 81

Array 2

3

Auxiliary Array

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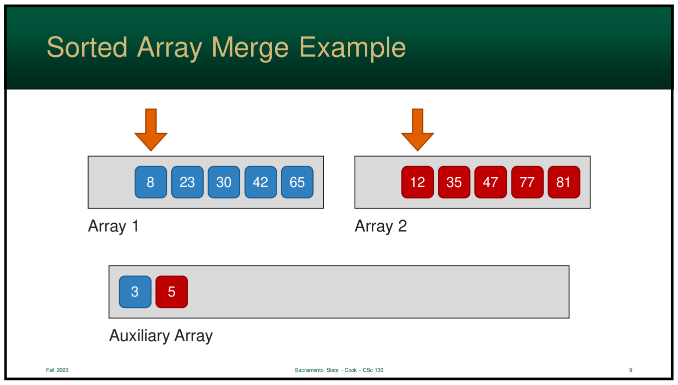

Sorted Array Merge Example

8 23 30 42 65

Array 1

12 35 47 77 81

Array 2

3 5

Auxiliary Array

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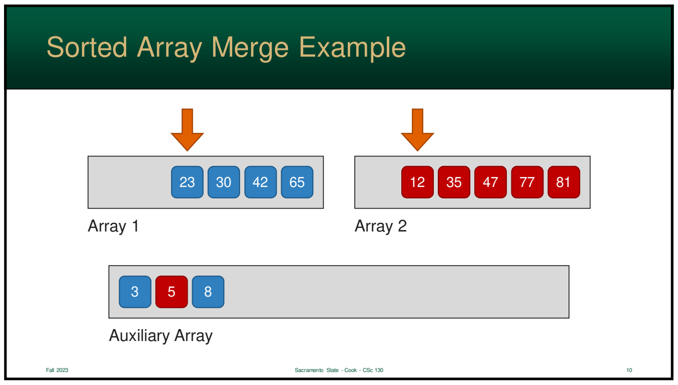

Sorted Array Merge Example

23 30 42 65

Array 1

12 35 47 77 81

Array 2

3 5 8

Auxiliary Array

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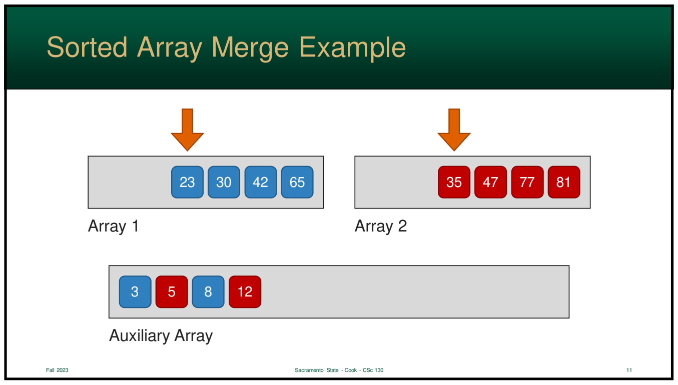

Sorted Array Merge Example

23 30 42 65

Array 1

35 47 77 81

Array 2

3 5 8 12

Auxiliary Array

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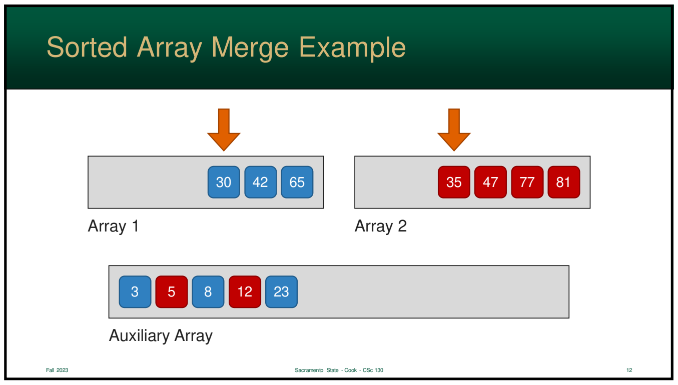

Sorted Array Merge Example

30 42 65

Array 1

35 47 77 81

Array 2

3 5 8 12 23

Auxiliary Array

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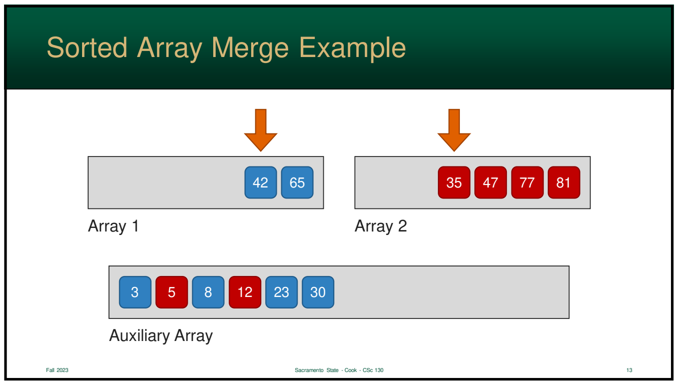

Sorted Array Merge Example

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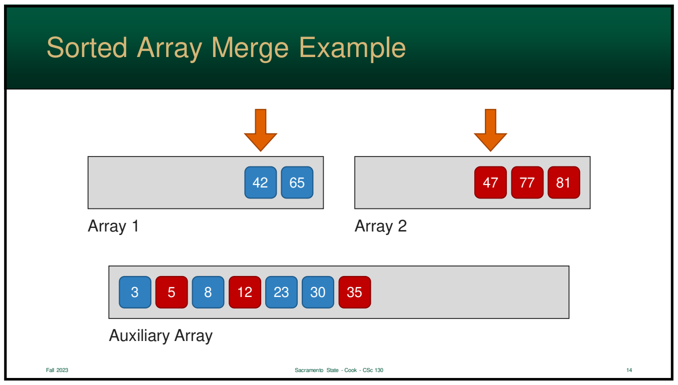

Sorted Array Merge Example

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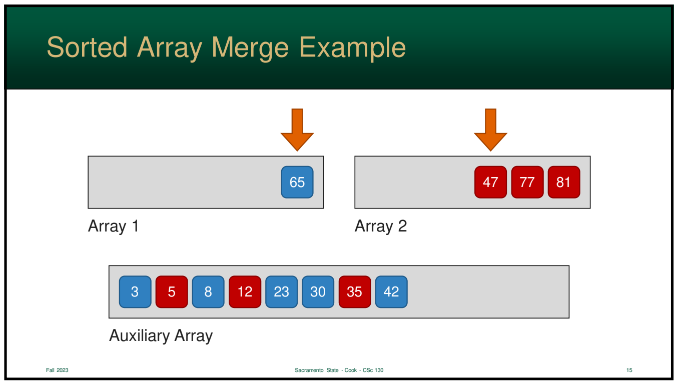

Sorted Array Merge Example

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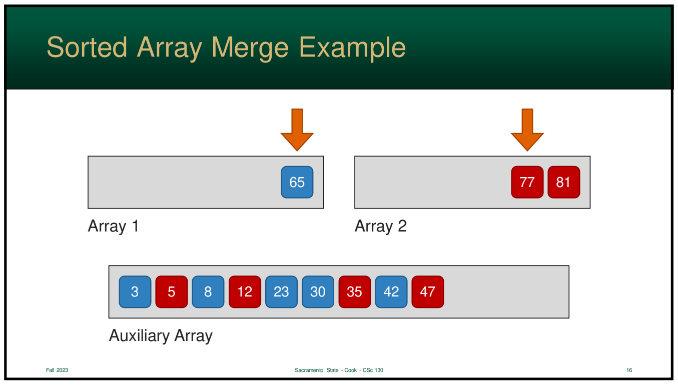

Sorted Array Merge Example

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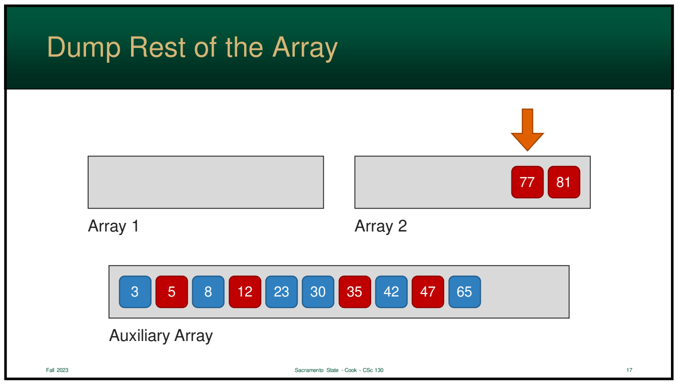

Dump Rest of the Array

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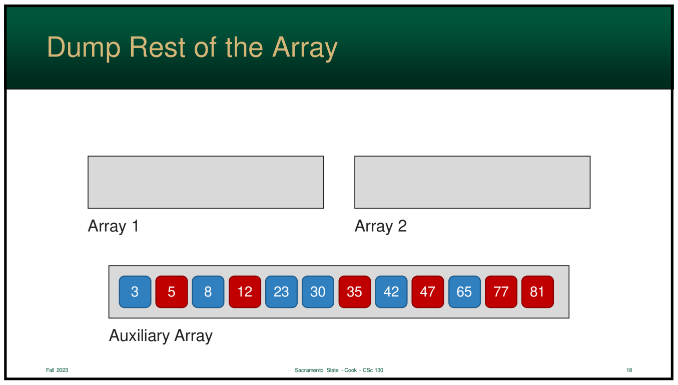

Dump Rest of the Array

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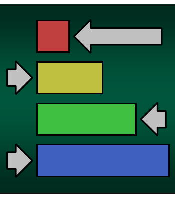

Merge Sort

Divide and conquer!

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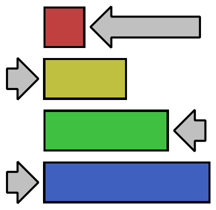
Merge Sort

- *Merge Sort* is a divide-and-conquer algorithm that cuts an array into smaller and smaller sublists until sorting them is arbitrary
- Invented by *John von Neumann* in 1945 *(19 BBW)*



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Merge Sort

- Because Merge-Sort defines a dividing the list into a list into smaller instances of itself, it naturally is solved using recursion
- Each recursive step cuts the list into two sublists until…
  - the list has 2 elements – arbitrary swap
  - the list has 1 element – which is, well, sorted

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Merge Sort

- As the recursion bubbles up, each sub list is merged using the algorithm we just discussed
- Divide-and-conquer algorithms ultimately result in O(n log n)
- Since an auxiliary array is required for the merge process, Merge-Sort, while fast, has O(n) auxiliary storage requirements

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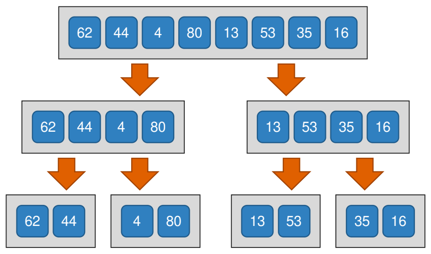
Merge Sort Example: Recurse down



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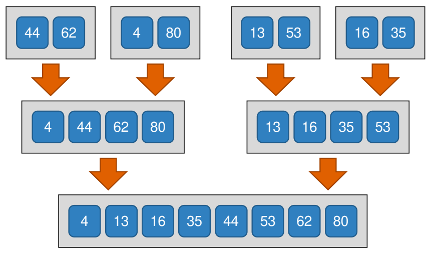
Sort Merge Sort Example: Merge Up



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Merge Sort Summary

Merge Sort	
Time Average	O(n log n)
Time Best	O(n log n)
Time Worst	O(n log n)
Auxiliary space	**O(n)**
Stable	Yes – Equal element order preserved
Online?	Yes – New data → new sublist

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Quick Sort

Oh, I am getting dizzy....

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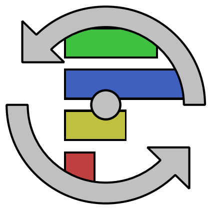
Quick Sort

- *Quick-Sort* is a divide-and-conquer algorithm that rotates values around a *pivot*
- Invented by *C. A. R. Hoare* in 1959 *(5 BBW)*
- Even faster than both Merge Sort and Heap Sort
- ... but has a weaknesses



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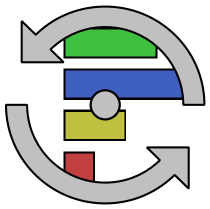
How it Works

- Like Merge-Sort, the array is broken down into smaller and smaller sub-lists
- However, before recursion
  - a value *p* is chosen in the sub-list as the *pivot* value
  - smaller items are moved before it
  - larger items are moved after it



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Choosing a Pivot

- Pivot can be any element in the sub-array
- …we need one actual value to compare
- This *pivot* is used to *partition* the values
- Different versions use different pivots
  - first item in the sub-array
  - end item in the sub-array
  - the midpoint of the sub-array
  - random value in the sub-array

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Partitioning the Values

- After the pivot *p* is selected, all elements are moved
- Two, separate, loops move through the elements and swaps elements less than/greater than the pivot
- The result is...
  - sub-array **L** contains items less than *p*
  - sub-array **G** contains items greater than *p*

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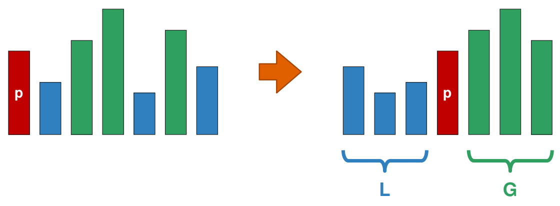
Partitioning (pivot is the first item)



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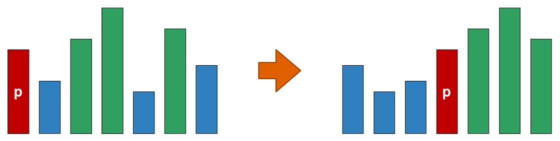
Partitioning the Values

- Note: neither **L** or **G** is sorted yet
- These will be called recursively by Quick-Sort
- Moving the elements, in-place, can look a tad ugly code-wise, but the logic is straight forward



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Partition Algorithm

- The sub-lists are stored in the original array – so there's no auxiliary storage
- The algorithm maintains two pointers
  - first moves left to right and keeps track of the values that are **too big**
  - second moves right to left and keeps track of the values that are **too small**
- Each moves independently

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Partition Algorithm

- First move the **Too Big** pointer until a value is found that is bigger than the pivot
- Then move the **Too Small** pointer until a value is found that is smaller than Pivot
- Then, these values are swapped
- When the two pointers collide, we are done

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Example Partition

- *In this example*, we pivot at the start of the array
- Any value can be used...
  - but it will have to be swapped to the start before the algorithm runs
  - this "saves" the pivot for later

42	8	12	77	65	30	47	52	9	35	23

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Quick Sort Algorithm

```
while (tooBig < tooSmall)
{
   while (array[tooBig] <= array[pivot])
   {
       tooBig ++;
   }

   while (array[tooSmall] > array[pivot])
   {
       tooSmall --;
   }

   if (tooBig < tooSmall)
   {
      //swap array[tooBig] and array[tooSmall]
   }
}
//swap array[tooSmall] and array[pivot]
//Recurse QuickSort on both L and G
```

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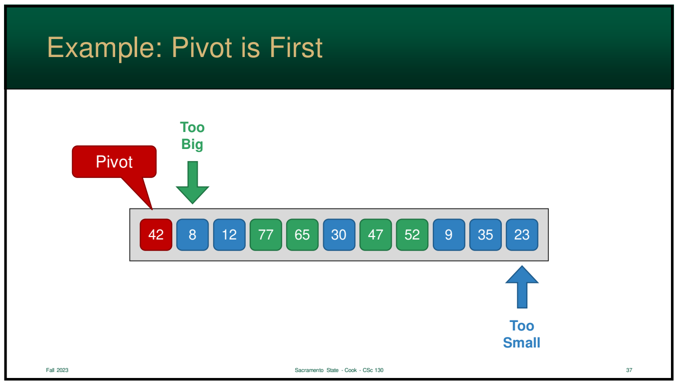

Example: Pivot is First

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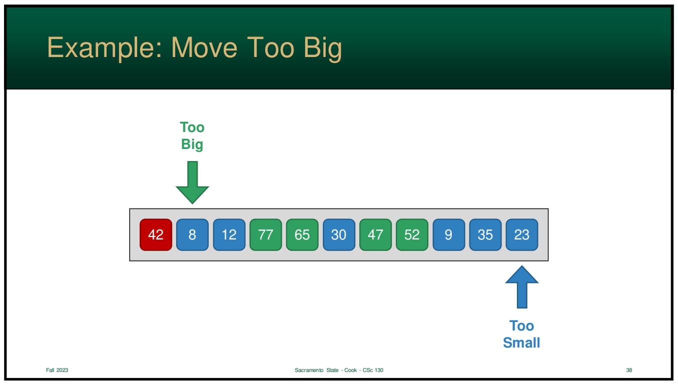

Example: Move Too Big

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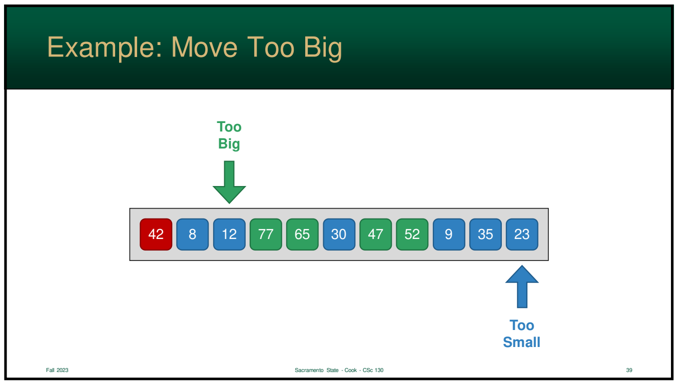

Example: Move Too Big

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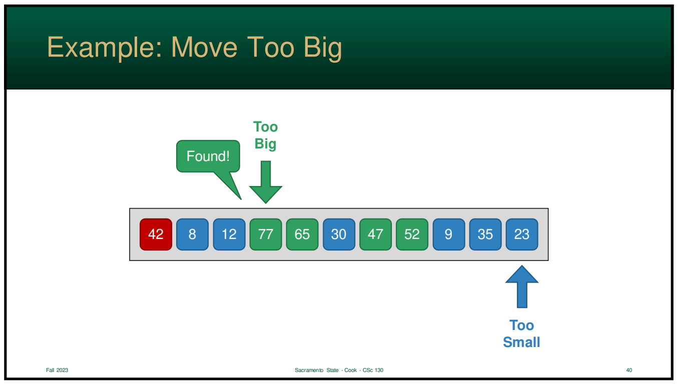

Example: Move Too Big

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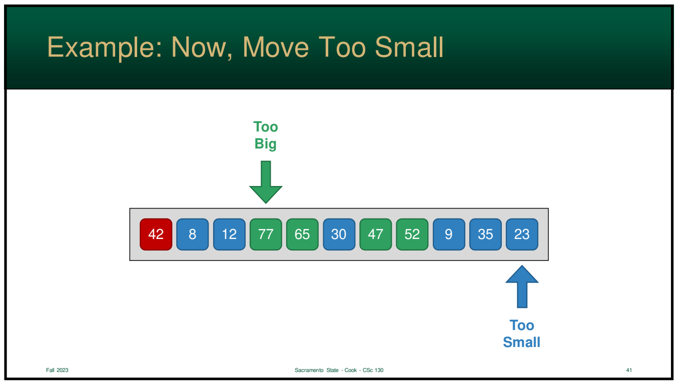

Example: Now, Move Too Small

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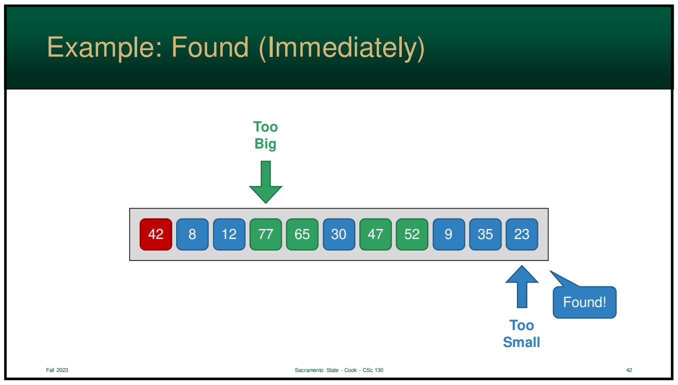

Example: Found (Immediately)

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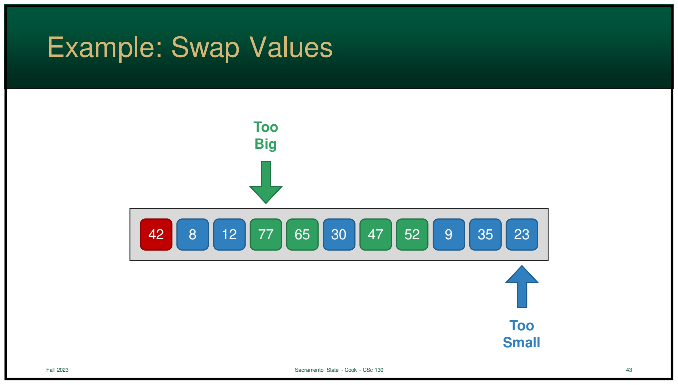

Example: Swap Values

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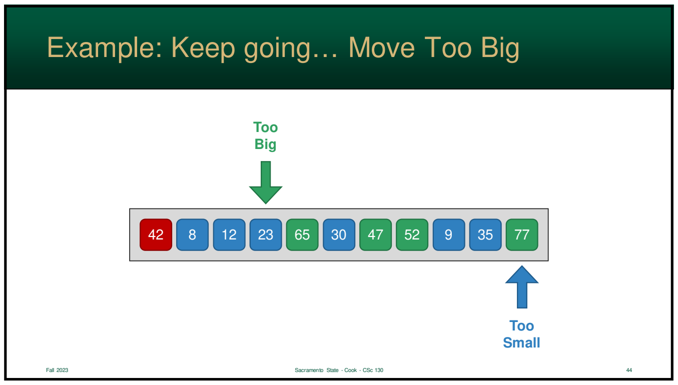
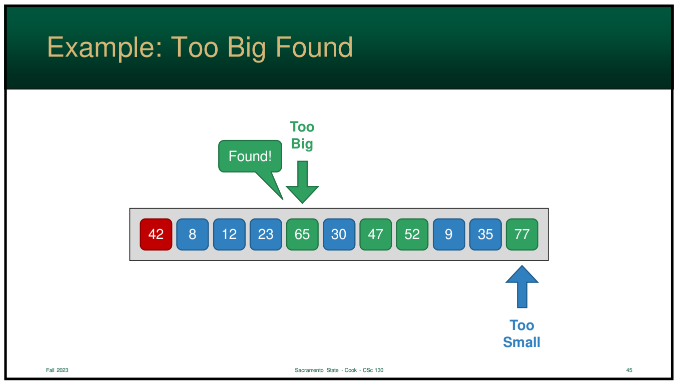


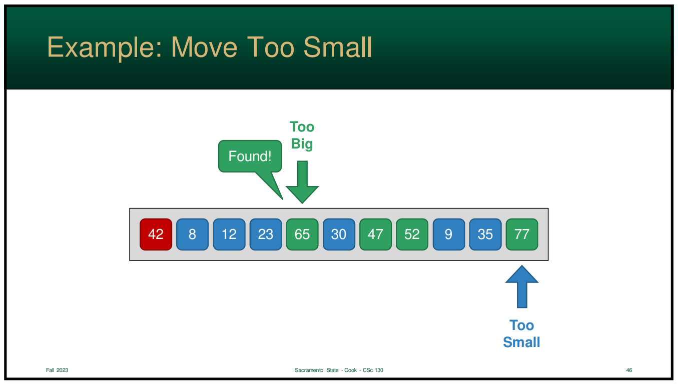
Example: Keep going… Move Too Big

44



Example: Too Big Found

45



Example: Move Too Small

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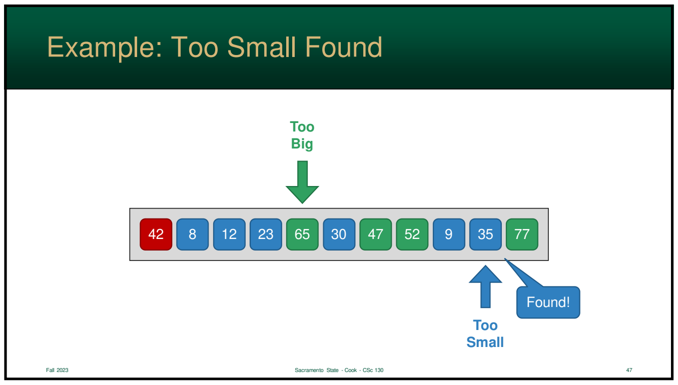

Example: Too Small Found

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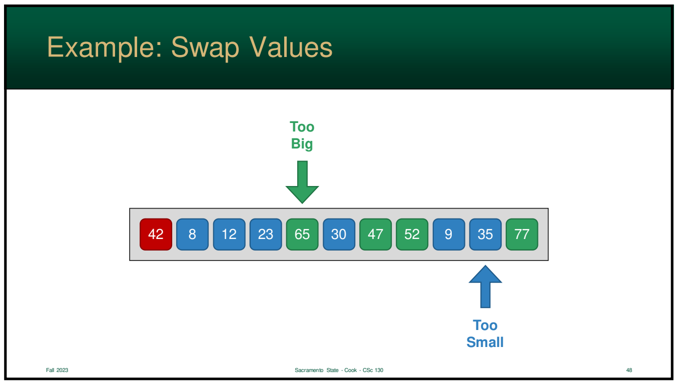

Example: Swap Values

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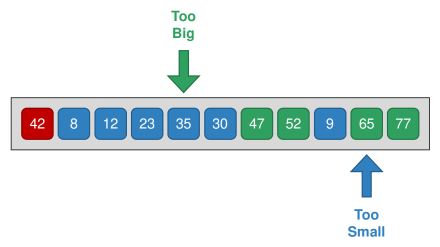
Example: Keep going… Move Too Big



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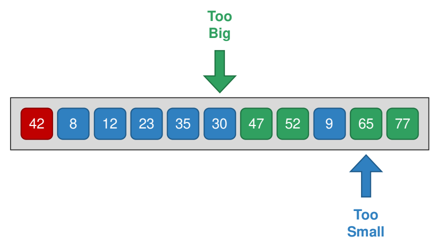
Example: Keep going… Move Too Big



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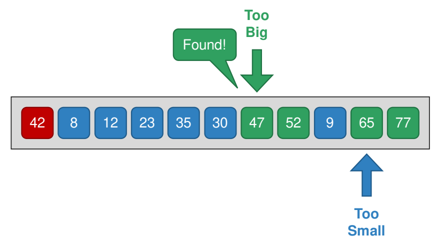
Example: Too Big Found



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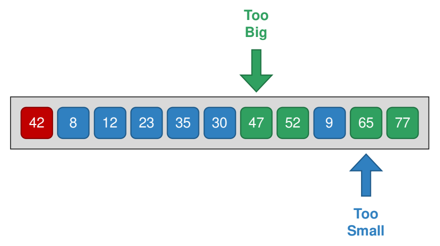
Example: Move Too Small



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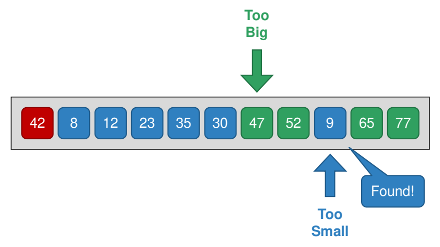
Example: Too Small Found



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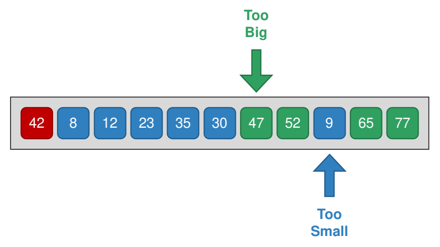
Example: Swap Values



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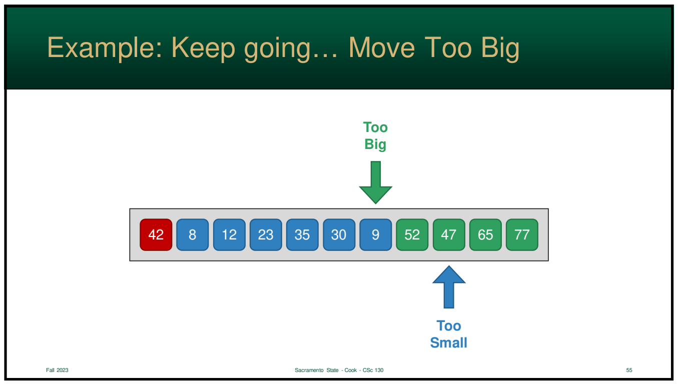

Example: Keep going… Move Too Big

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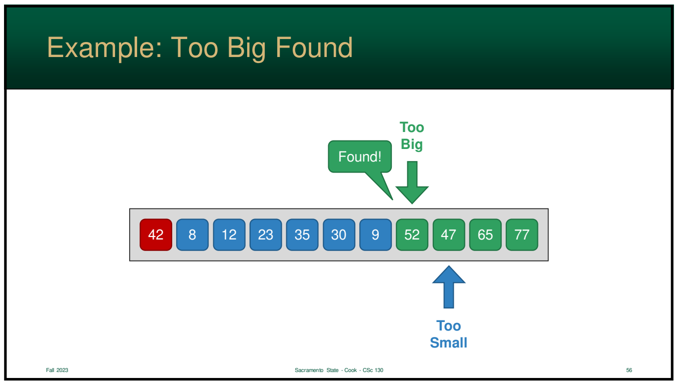

Example: Too Big Found

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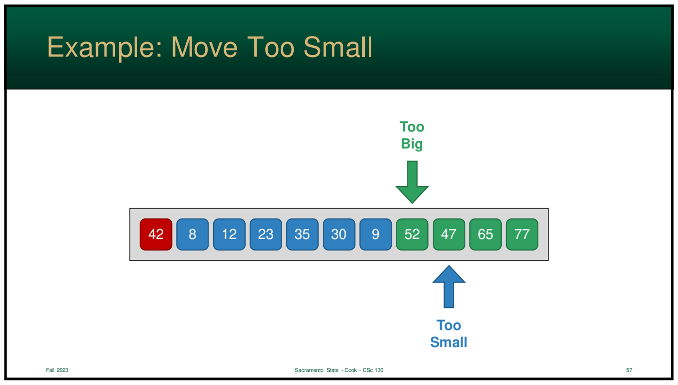

Example: Move Too Small

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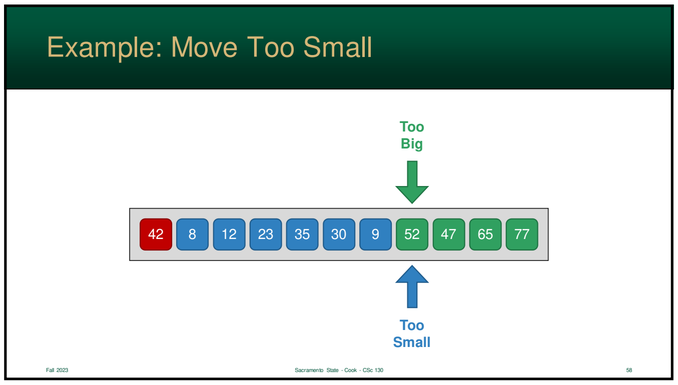

Example: Move Too Small

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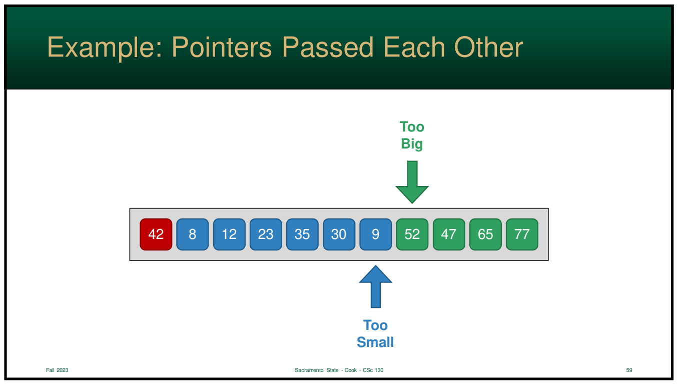

Example: Pointers Passed Each Other

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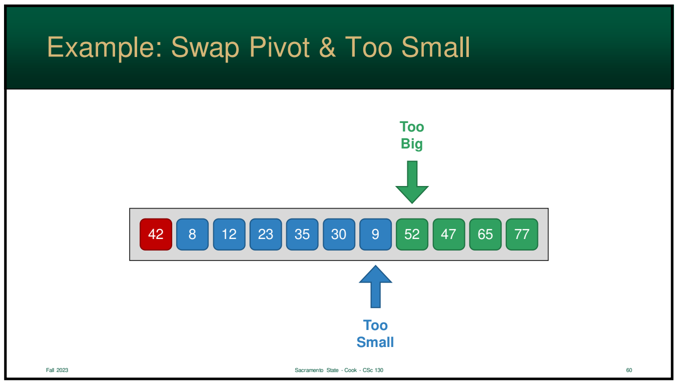

Example: Swap Pivot & Too Small

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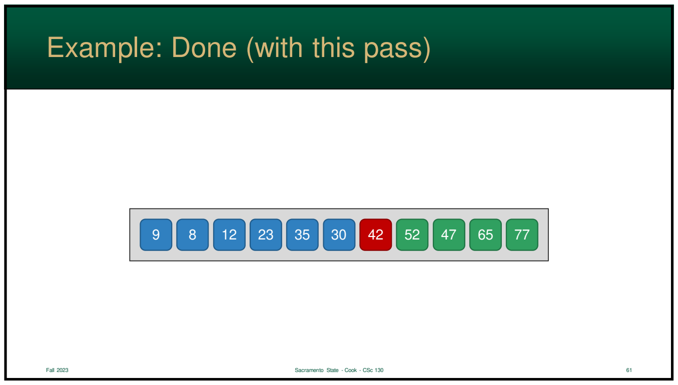

Example: Done (with this pass)

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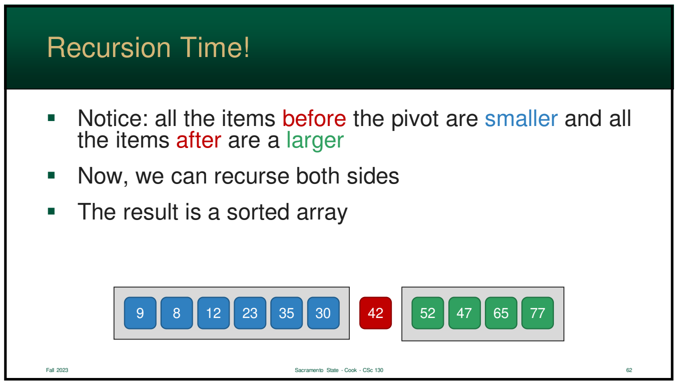

Recursion Time!

- Notice: all the items before the pivot are smaller and all the items after are a larger
- Now, we can recurse both sides
- The result is a sorted array

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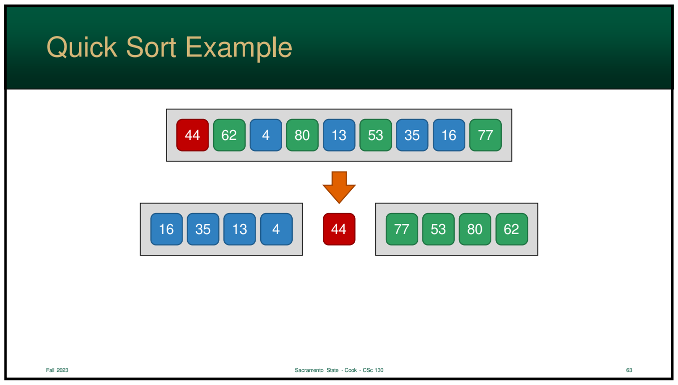

Quick Sort Example

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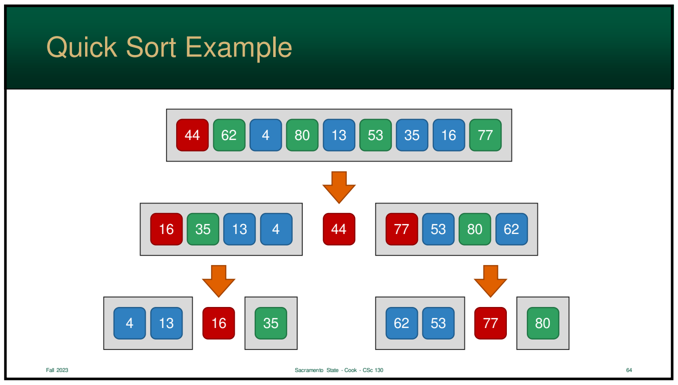
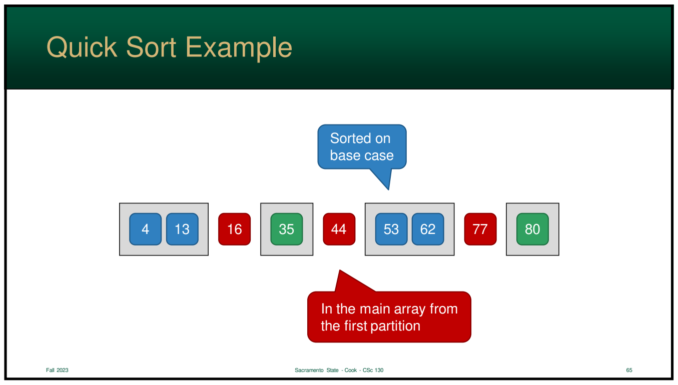


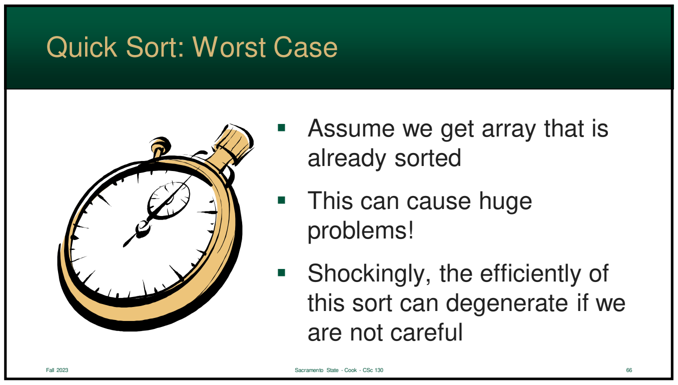
Quick Sort Example

64



Quick Sort Example

65



Quick Sort: Worst Case

- Assume we get array that is already sorted
- This can cause huge problems!
- Shockingly, the efficiently of this sort can degenerate if we are not careful

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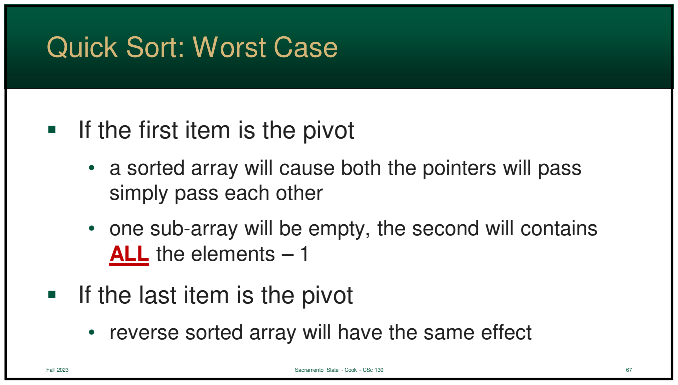

Quick Sort: Worst Case

- If the first item is the pivot
  - a sorted array will cause both the pointers will pass simply pass each other
  - one sub-array will be empty, the second will contains **ALL** the elements – 1
- If the last item is the pivot
  - reverse sorted array will have the same effect

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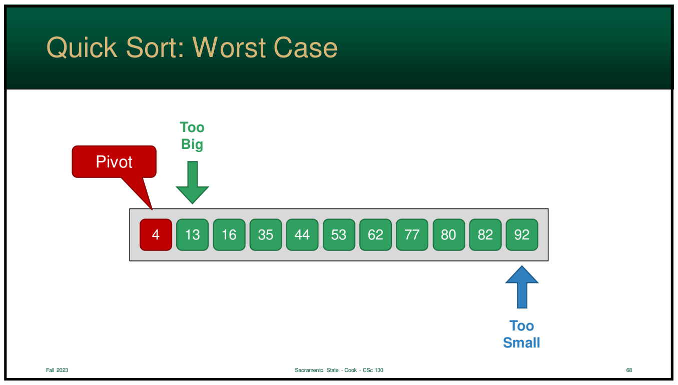

Quick Sort: Worst Case

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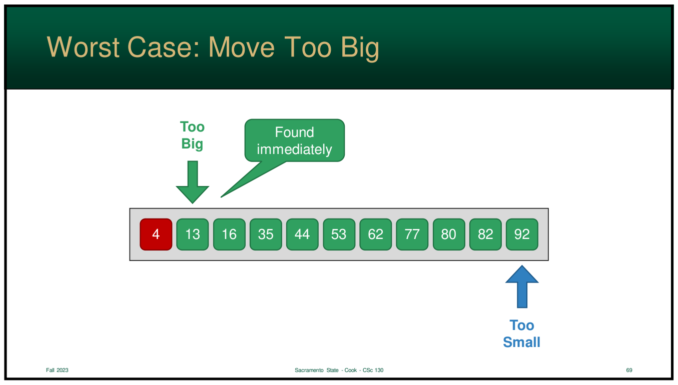

Worst Case: Move Too Big

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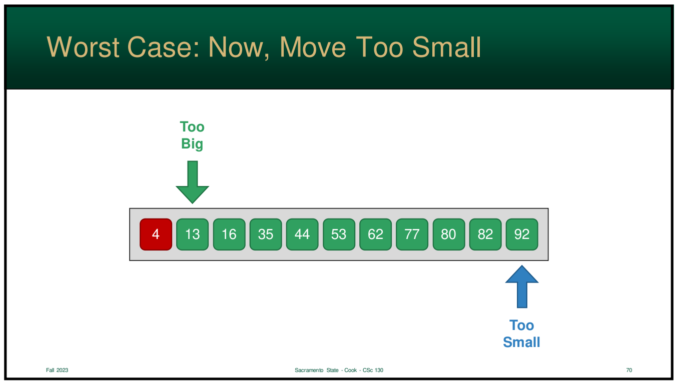

Worst Case: Now, Move Too Small

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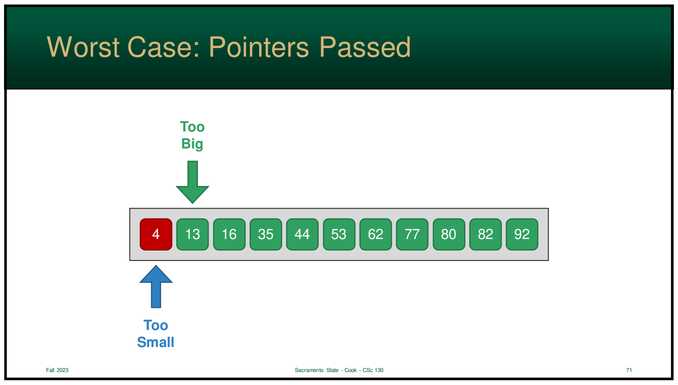

Worst Case: Pointers Passed

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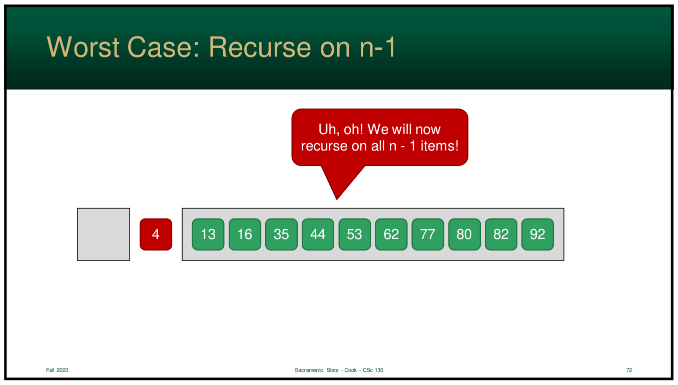

Worst Case: Recurse on n-1

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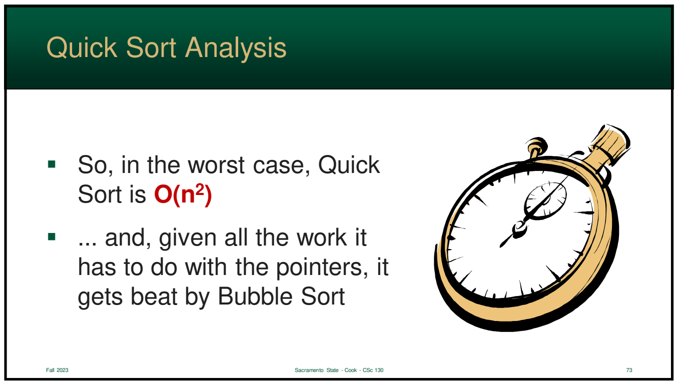

Quick Sort Analysis

- So, in the worst case, Quick Sort is **$O(n^2)$**
- ... and, given all the work it has to do with the pointers, it gets beat by Bubble Sort

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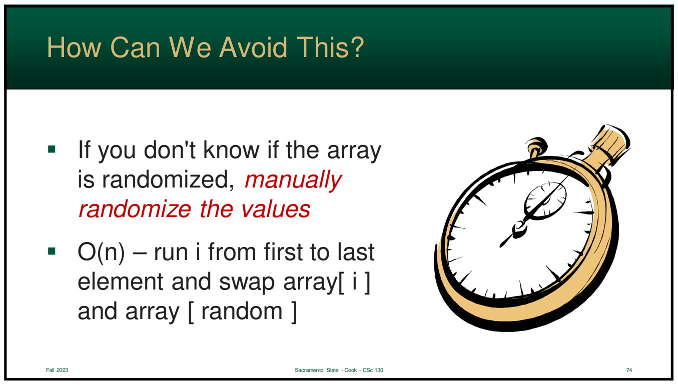

How Can We Avoid This?

- If you don't know if the array is randomized, *manually randomize the values*
- O(n) – run i from first to last element and swap array[i] and array [random]

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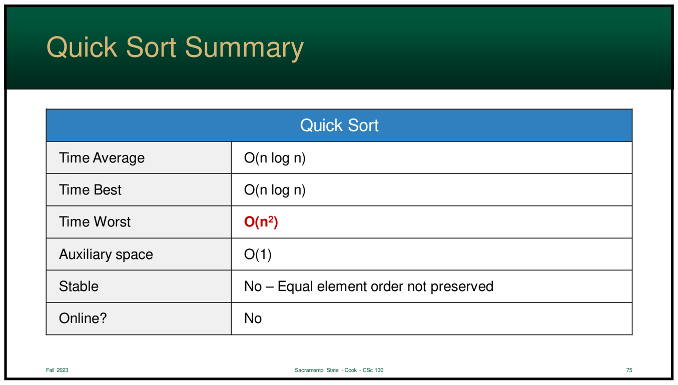

Quick Sort Summary

Quick Sort	
Time Average	O(n log n)
Time Best	O(n log n)
Time Worst	**$O(n^2)$**
Auxiliary space	O(1)
Stable	No – Equal element order not preserved
Online?	No

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